

CASE 3

A Picture Is Worth a Thousand Words

PRE-CASE EXERCISES

Please consider the following situation: You are attempting to enlarge a picture using the math department's copier. The copier is designed so that when the enlarger is set at 200%, the area of the copy is twice the area of the original photo, while if the "enlarger" is set at 75%, the copy's area is $\frac{3}{4}$ of the original's area. Solve the following problems, writing out your solutions in complete sentences.

1. If the original picture's dimensions are 4 inches by 6 inches, what are the dimensions of the copy when the enlarger is set at 100%? Does your answer depend on the orientation of the picture in the copier? Explain.
2. The copy from above is now placed back in the copier with the enlarger now set at 150%. What are the dimensions of the new copy? To what would we have to set the enlarger in order to get the same copy from the original photo?
3. You decide you want to reduce the picture to fit onto a 3×5 index card. Is it possible to get a copy the dimensions of which are *exactly* 3 inches by 5 inches? If so, how should we set the enlarger? If not, how should we set the enlarger so that the longer side of the copy is *exactly* 5 inches in length?

THE CASE

"Harry, Harry! Quiet down and get to work!" Mr. Itak bellowed to one of his sophomores. He put his hands to his hips as he frowned. The groups in his geometry class were getting noisier, and he was getting nervous. Out of the corner of his eye he spotted Dr. Lanois, his principal, sitting in the

back and furiously scribbling. Of all the days for her to pick for a surprise observation! Today he had given his class a group assignment asking them to use ratios. As he began the class, Mr. Itak handed each student a hand-drawn worksheet (see Figure 3.1).

He approached the students in Harry's group, who were playing with their rulers. "What work have you done so far?" he snapped to get their attention.

"What?" responded Harry, putting his ruler down. Mr. Itak considered Harry to be a bellwether student in the class, so Mr. Itak often asked him questions as a gauge. If Harry understood something, then most of the others would. Harry peered from under his baseball cap. "Well, we're stuck."

"Let me see what you've done." Reluctantly, Harry handed his group's paper to Mr. Itak and slumped in his seat (see Figure 3.2). Kate and Jahmal, his other groupmates, sat motionless and silent.

As he scanned their collective work, he couldn't believe it. "Are you telling me that in the real world these steps are only a quarter of an inch high?"

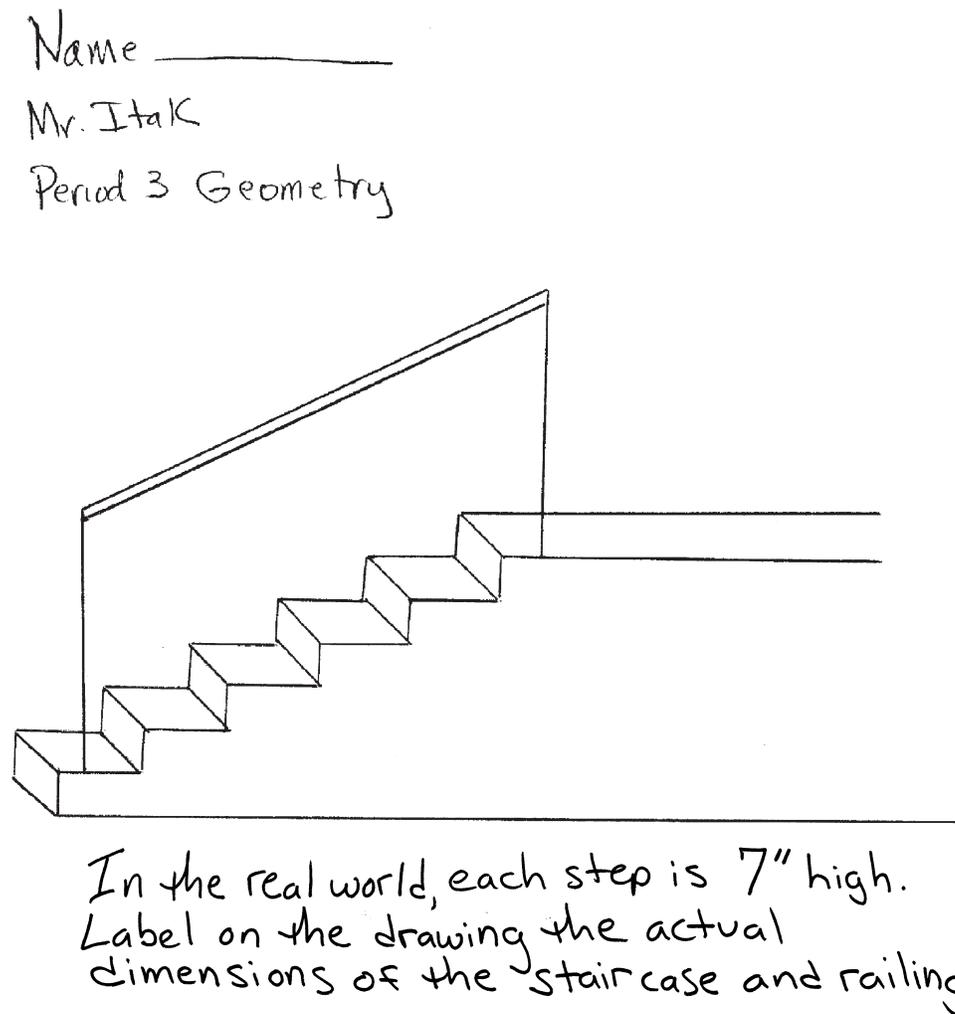
"That's what the ruler said," Harry quipped.

Exasperated, Mr. Itak gave the sheet back to Harry and took out his wallet. "Here is a picture of my wife. She's not really this small. She's really bigger. If I measure her nose in this picture to be a fourth of an inch, does it mean her nose is really one-fourth of an inch long?"

The group chuckled, "Oh, I see," said Harry, pulling his chair closer to the table.

"Now, can you find out how long the handrail is in the real world?"

Harry aligned his ruler on the picture. "Well, the ruler says it's about 3 inches, so it's really about $9\frac{3}{4}$ inches," he said happily.

Figure 3.1 Mr. Itak's Drawing

Mr. Itak crossed his arms and sighed deeply. He tried to stay calm. Why was his class having such difficulty?

Geometry at Kensington

Mr. Itak was starting a geometry unit on similar triangles, something he had done almost every year for the past 25 years at Kensington High School. Over the past few years, he had become increasingly concerned that his students didn't understand the basic concepts. On tests they often forgot how to solve equations, and many would forget procedures such as the distributive law. Now, with the advent of the state competency test, such lapses were no longer a minor irritation. The local paper had reported that the district's competency scores were lower than comparable communities.

With the recent reissue of the NCTM Standards, Mr. Itak decided that he wanted to get his students more involved in real-world applications. He had seen some projects in old copies of *The Math Teacher* that asked students to use ideas such as area to construct floor plans. He decided to design some real-world applications himself this year.

The 25 students in his sophomore geometry class were demographically typical of the student body of 1,200 at Kensington. Most of his students were White and middle class, but he also taught a few African-American students who had been bussed in from the nearby Middletown. Unlike the upper-level math classes at Kensington—which consisted mostly of boys, and were dominated by them—the lower-level classes had nearly equal numbers of boys and girls. Recently the math department had been trying to encourage girls to

take upper-level math classes by reassessing the way lessons were taught and by exposing the girls to role models in mathematical fields. When it came to lower-level classes, the math chairman often said that the teacher's job was to teach students enough math so they might get into a junior college. But Mr. Itak doubted that more than half of this class ever would attend college at all.

It was January and the class had just finished congruence. Looking at the rest of his curriculum, Mr. Itak decided his students needed some work on ratios and proportions before they could move into the similarity unit. After a brief review of the algebra of ratios and proportions, he wanted to give his students a chance to apply the algebra. Using real-world objects that students could relate to might be an interesting approach that could lead him to similarity. *All of these students have seen stairs,*

he thought. *The real-world connection is obvious.* So, in between grading papers in a prep period, he quickly drew a picture of a set of stairs for a group project. He did not have much time, so he just used a ruler and roughly sketched a picture of a set of stairs with a railing and wrote some directions underneath.

Harry's Staircase

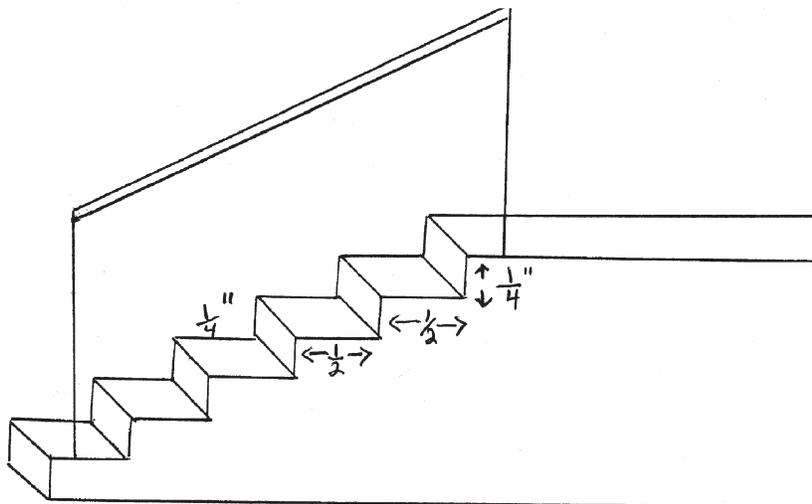
Mr. Itak could not understand Harry's answer. Harry began to explain.

"It's like, the difference thing. You, like, added $6\frac{3}{4}$ to go from $\frac{1}{4}$ to 7 so I like, added $6\frac{3}{4}$ to the 3 to get around $9\frac{3}{4}$."

"You added?" Mr. Itak pulled up a chair. "Have you ever seen a handrail that is only $9\frac{3}{4}$ inches long?"

Figure 3.2 Harry, Kate and Jahmal's Work

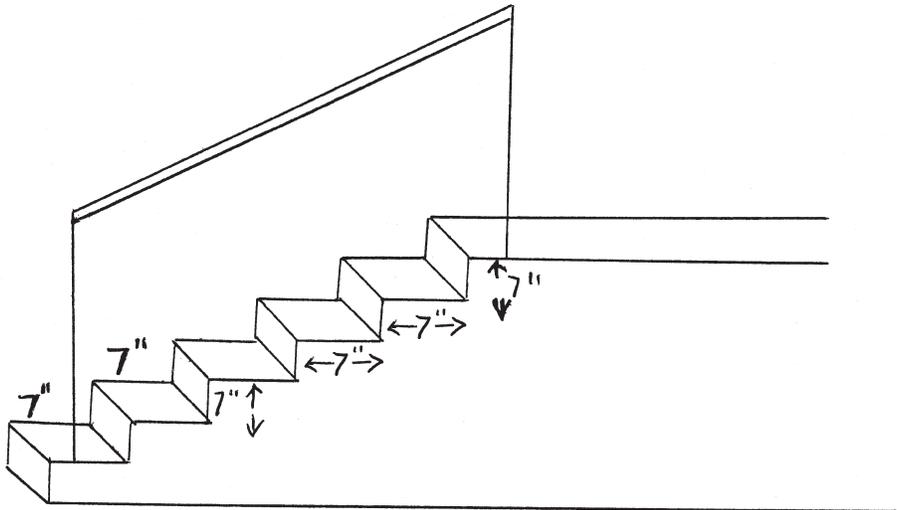
Name ~~Harry's~~ Kate's Jahmal's
Mr. Itak
Period 3 Geometry



In the real world, each step is 7" high.
Label on the drawing the actual
dimensions of the staircase and railing.

Figure 3.3 Maggie's Work

Name Maggie
 Mr. Itak
 Period 3 Geometry



In the real world, each step is 7" high.
 Label on the drawing the actual
 dimensions of the staircase and railing.

"It could be one of those dollhouses," Harry smirked.

"No, no, look at the drawing! This is a drawing of a real staircase," Mr. Itak said as he pointed to the drawing. "This is not an addition thing."

"I know," spoke up Kate, who had been silent in the group so far. Kate was usually quiet, particularly in groups, and Mr. Itak considered her an average student. "It's ratios. I don't know how, but it's ratios."

"Ah ha!" Mr. Itak exclaimed. "Ratios! Now, think about it."

Worried that other groups might share similar confusions, Mr. Itak stood and spoke loudly to the class. "Remember the drawing *represents* the real world! The measurements you are looking for aren't the ones in the drawing. You want to find the measurements in the real world. Remember

ratios?" He raised his eyebrows. Many of the students nodded and continued their work. He headed toward Maggie's group, the group nearest to Dr. Lanois. He hoped that at least they had the right answer.

Dr. Lanois

As Mr. Itak approached Maggie's group, Dr. Lanois looked up from her notes and smiled lightly. She was hired last year for her progressive ideals, which didn't sit well with most of the senior faculty. They had seen reforms come and go, and expected her to do the same. She was young and Mr. Itak supposed that her doctorate in education from a local Ivy League school impressed the school board. Because her background was in administration and she was an English teacher, Mr. Itak had

little in common with her. He never spoke with her about the sorts of things he did or even wanted to do in his class. The previous principal, Joe Redding, was a personal friend of Mr. Itak and a former math teacher. He talked often with Joe about teaching, and Joe always let him know beforehand if he was going to observe. Dr. Lanois's unannounced presence made Mr. Itak uneasy, and her constant note taking made him even more nervous.

Maggie's Staircase

"Maggie, pay attention." Maggie was a good student—she had Bs in almost all of her classes, including geometry. Mr. Itak could not believe her group wasn't getting this. "Let me see your group's work." Maggie handed the paper to him (see Figure 3.3).

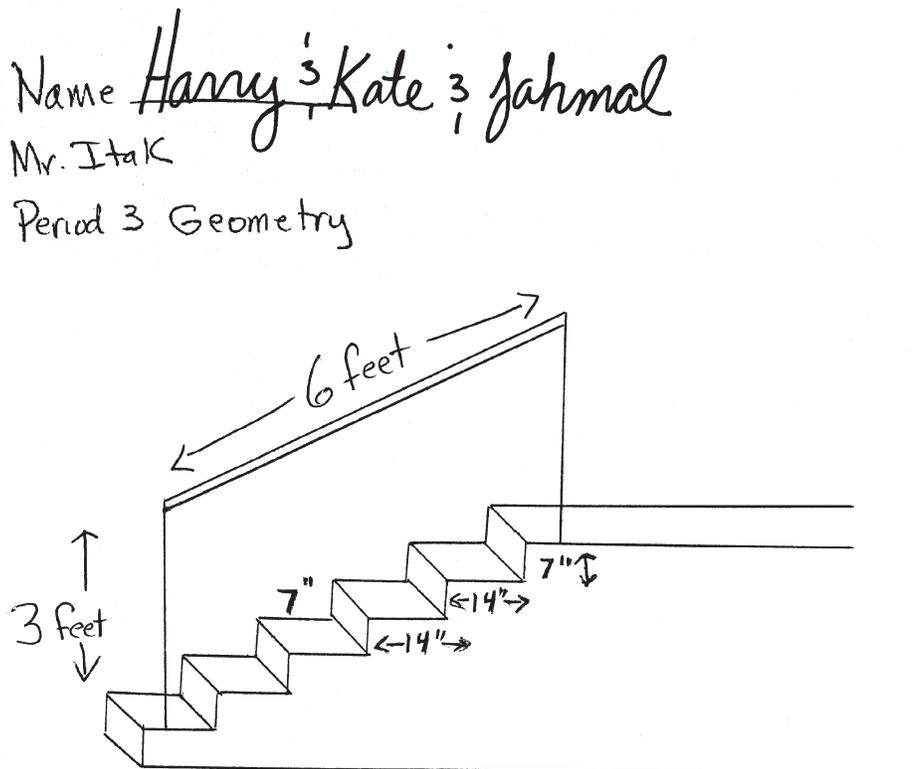
"Maggie, are these steps just as tall as they are wide in the drawing?"

"No," Maggie replied as she reached for her ruler, "they are $\frac{1}{4}$ inch high and $\frac{1}{2}$ inch wide in the drawing."

"Then why are you saying they are just as tall as they are wide in the real world?" Mr. Itak was getting worried. "Think about it. You can do it. If $\frac{1}{4}$ inch gives you 7 inches, how much will $\frac{1}{2}$ of an inch give you?" Maggie smiled. She looked confident now. Mr. Itak glanced at the rest of Maggie's group, who were watching Maggie busily write. "Make sure you explain it to the others, Maggie."

In a last-ditch effort, Mr. Itak walked back to Harry's group. They had added some measurements to their work, which they showed him quite proudly as he approached (see Figure 3.4).

Figure 3.4 Further Work from Harry's Group



In the real world, each step is 7" high.
Label on the drawing the actual
dimensions of the staircase and railing.

“Look guys, each step is correctly labeled as being 7 inches by 14 inches. But how can the height of the railing be 3 feet?”

“Well,” said Harry, “we used ratios. One-fourth is 7. One-half is twice $\frac{1}{4}$ so the $\frac{1}{2}$ is 14. So, the step is actually 14 inches wide. Then we got the height of the handrail. That’s $1\frac{1}{2}$ inches in the picture. So, we doubled that to get 3. It can’t be 3 inches, so it must be 3 feet. You know, real world like you said.”

The bell rang. “Okay, time to pick up,” Mr. Itak exclaimed. “Put your stuff away. No homework tonight. But I want you to think about what we have been doing. You should be using ratios in this.”

The students filed out past a discouraged Mr. Itak as Dr. Lanois approached him. “Tough class,” she said smiling.

“Well,” Mr. Itak began, “they are a good group

of kids. They tried and I think that some of them are getting it. But I think—”

“Well, I have to run,” she interrupted, glancing at her watch. “We need to set up a postevaluative conference to talk about this. See my secretary and we’ll schedule one.”

Great, Mr. Itak thought. This is all I need. I should have stayed with the book.

QUESTIONS

1. What do you think of Mr. Itak’s idea to review ratios?
2. How much do students benefit from real-world applications when their basic skills are weak?
3. What, exactly, was confusing the students?
4. How directive should a teacher be in a problem-solving situation such as this?