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Examining the Relationship Between Beliefs and Goals in Teacher Practice

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This article presents a detailed analysis of how teacher beliefs interact with goals and influence the moment-to-moment actions of teaching. The beliefs, goals and instructional practice of two secondary mathematics teachers were examined as each conducted an algebra lesson. We discuss how specific beliefs organized to influence the selection and prioritization of goals that then influenced the actions of each teacher. Fine-grained analysis of classroom video and teacher interviews revealed that particular collections of beliefs become apparent when there is a shift in the teacher's goals. Exploring the relationship between teacher beliefs and goals at this level of detail allows for the investigation of the mechanisms of the teaching process. Implications of this research for teacher education and professional development are also discussed.

1. INTRODUCTION

The connection between teacher beliefs and practice has been well established (see e.g., Calderhead, 1996; Cohen, 1990; Ernest, 1989; Thompson, 1984, 1992). However, the nature of that connection and its development, as well as the specifics of what transpires in the moment-to-moment action of teaching, have scarcely been examined. Many issues and questions need further investigation. For instance, how do beliefs influence the decisions teachers make during classroom interactions? Are certain beliefs particularly influential during teacher–student interactions? If goals play an important role in the teaching process, how do beliefs influence the formulation of those goals? As part of a larger project to model the teaching process (Schoenfeld, 1998, 2000), this study initiates the substantive investigation of these questions. Specifically, we make two claims in this article and provide some empirical warrants for them. First, we argue that beliefs play a central role in a teacher's selection and prioritization of goals and actions. Second, we claim that when there is a shift in goals during teaching, the teacher's beliefs are likely to be apparent. In the study of the beliefs, goals and practices of two teachers reported here, we begin to unravel the complexity of the relationships among these aspects of teaching.

Issues related to beliefs and goals are significant for several reasons. First, we contend that it is important for researchers to come to a fuller, more detailed understanding of the significant influence of beliefs on the teaching process than is currently available. Being able

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to identify and describe the *mechanisms* underlying the influence of beliefs on instructional interactions would deepen and enrich our understanding of the teaching process.

Second, uncovering how beliefs influence specific aspects of the teaching process will contribute to our overall understanding of teaching. The role of goals as a component of the teaching process has been researched to some extent. However, research on goals and research on beliefs have, in the past, been quite isolated from one another. The current study begins to explore the connections between these two areas of research.

Third, having an understanding of and a specific language with which to describe the elements that influence the teaching process will promote discussions of issues related to beliefs at a level of detail not previously possible. Such a language could facilitate discussions about the interaction of beliefs and practices. This kind of discussion could be a productive component of teacher preparation and professional development programs.

This article is organized into five sections. First, we present a brief overview of literature relevant to this study. Second, we situate this study as part of the ongoing development of a theoretical model of teaching-in-context (Schoenfeld, 1998). The third section outlines the framework and methods we use for examining the connections among beliefs, goals, and practice. There we present our claims, describe how we developed our methods in conjunction with the study, and discuss how these methods enabled us to capture the relationship between beliefs and goals. In the fourth section, we present two examples of classroom practice to illustrate our claims. The final section outlines implications and some future directions for research.

2. RELEVANT LITERATURE

This study draws upon different areas of the psychological and educational literature about beliefs, with a focus on work in teacher beliefs from the field of mathematics education. We also draw upon research devoted to goals and to the relationship between goals and actions.

2.1. Teacher Beliefs and Practice

Beliefs have been described as a "messy construct" with different interpretations and meanings (Pajares, 1992), yet there is some convergence of views found in the literature. Current definitions of beliefs found in the mathematics education literature focus primarily on how teachers think about the nature of mathematics, teaching and learning. In this context, beliefs are defined as conceptions, personal ideologies, world views and values that shape practice and orient knowledge (Ernest, 1989; Thompson, 1984, 1985, 1992).

Research has documented that the beliefs teachers hold related to mathematics, teaching and learning have a significant influence on their instructional practices (Calderhead, 1996; Cohen, 1990; Stodolsky, 1988; Thompson, 1992). This influence on practices plays out in interesting ways. In some cases, research has found that professed beliefs about mathematics, teaching and learning were consistent with what was found in observations of classroom practice. For example, Thompson (1985) described a teacher named Kay who viewed mathematics as more of a "subject of ideas and mental processes than a subject of facts" (p. 288). She also viewed the study of mathematics as one of "discovery and verification of ideas." Kay's professed views about mathematics were consistent with her professed views about teaching students and with Thompson's observations. According to Thompson (1985),

> She frequently encouraged the students, in a rather persuasive tone, to guess, conjecture, and reason on their own, explaining to them the importance of these processes in the acquisition of mathematical knowledge. (p. 289)

Thompson attributes Kay's consistency between what she says she believes and what she does in the classroom to her being reflective about her practice.

Researchers have also documented the inconsistencies between professed beliefs and instructional practice. Thompson (1984) described a teacher named Lynn who professed to believe that mathematics instruction should encourage students to ask questions and participate actively in class discussions. However, Thompson observed that Lynn's practice primarily consisted of lectures followed by routinized seatwork. These practices, which severely restricted student participation, interaction and the opportunity to ask questions, were inconsistent with Lynn's professed beliefs. Thompson (1984) attributed this inconsistency to Lynn's underlying conceptual view of mathematics teaching and learning. She held beliefs such as, "mathematics instruction is the means for transferring information from teacher to student" (p. 117) and learning beliefs such as, "students learn mainly by attentively watching the teacher demonstrate procedures and methods for performing mathematical tasks and by practicing those procedures" (p. 117).

Cohen (1990) provided a similar case analysis of a teacher named Ms. Oublier. Ms. Oublier believed she was implementing reform mathematics practices. However, Cohen claimed that she had in fact maintained very traditional teaching practices. Upon closer examination, Cohen identified beliefs that were in many ways antithetical to the constructivist views of mathematics teaching and learning that undergird recent mathematics reform efforts. Cohen described the ways in which these beliefs were exerting considerable influence on her practice.

The case studies of Cohen (1990) and Thompson (1984, 1985) provide evidence that beliefs affect practice in complex ways. They describe how a teacher's set of beliefs can have an influence on the overall nature of practice. They also explain that what teachers profess to believe and what they actually do in the classroom may or may not be consistent. Although these studies contribute a great deal to our general understanding of beliefs and practice, what remains unexplored are the details of how those beliefs inform practice, particularly in the formulation of goals and in the moment-to-moment actions of a teacher in a classroom.

2.2. Teacher Goals

The second line of research that informs this study centers on goals. Research on teacher goals typically focuses on their relation to teacher planning or their function in an information processing model. The first type of work tends to focus on teachers' long-range, general goals for student learning or curriculum development (Clark & Peterson, 1986). The second type defines goals narrowly within the context of particular mental structures (Schank & Abelson, 1977). Although informative, neither of these perspectives on goals address the issues at the appropriate grain size or level of detail for the current study. However, an alternative view of goals is offered by Saxe (1991):

> Not only do individuals shape and reshape their goals as practices take form in everyday life, but they also construct goals that vary in character as a function of the knowledge they bring to practices ... Goals, then, are emergent phenomena, shifting and taking new form as individuals use their knowledge and skills alone and in interaction with others to organize their immediate contexts. (pp. 16–17)

Our research builds upon Saxe's characterization of goals as emergent phenomena. We believe that this definition is appropriate for the study of goals and teaching for several reasons. The work of Clark and Peterson (1986) focuses on a level of detail which is too broad for the issues that concern us such as the moment-to-moment actions of teaching. The work of Schank and Abelson (1977) presumes that people have stable and pre-existing goals. In our study of teaching interactions, we need to allow for the existence of goals that arise in response to planned and unplanned events in the classroom; hence, Saxe's characterization is appropriate for our work.

We contend that beliefs shape how teachers perceive and interpret classroom interaction. Beliefs also influence the construction of their goals in response to those interactions. For example, suppose that in the course of a class discussion a student begins to incorrectly describe how to solve a problem. Believing that students learn by listening to the teacher, that teacher may decide to curtail the student's flawed presentation of the solution and proceed to present the correct solution. The teacher's goal of ensuring that a correct solution is presented to the students in the class is predicated in part on her beliefs about how students learn.

2.3. Goals and Beliefs

Surprisingly, very little has been written about the connections between goals and beliefs. There is a common understanding that beliefs and goals are related, but this connection has not been established empirically. Artificial intelligence is one area of research in which an attempt has been made to relate beliefs and goals. The work in this area examines their connection in the context of an information-processing model. An example of such a description comes from Schank and Abelson (1977):

> ... that in order to determine what goals an individual is likely to have at any given time, it is necessary to have an available set of beliefs about what an individual is likely to want in a given circumstance. (p. 119)

From their perspective, beliefs exist in the form of expectancy-rules and these rules test for a given situation. If that situation is present, predictions are generated about possible actions or feelings that will occur in response to the situation. From these predictions, goals are developed. However, Schank and Abelson's work assumes that most humans share, to varying degrees, a basic set of beliefs about how to behave and how others behave in a given situation. Given the work others have done on teacher beliefs and practice demonstrating the wide range of beliefs that may influence practice (Ernest, 1989;

Lerman, 1990; Thompson, 1992), this appears to be a huge assumption. In addition, the work of Schank and Abelson (1977) describes a relationship between beliefs and goals, but the nature of that relationship in the moment-to-moment interaction in a situation is not addressed. We would agree that people have sets of beliefs that organize their knowledge and influence the development of goals, however, what is unclear is how this plays out in the decision-making processes and actions of a teacher.

3. TEACHER MODEL BACKGROUND

In this section, we describe the research program of the Teacher Model Group (TMG) and how the current study contributes to the development of a comprehensive model of the teaching-in-context (Schoenfeld, 1998). We also describe the way in which instructional interactions are analyzed and represented in this model. This section gives the background information and context for our specific study, which is presented in the subsequent section.

The TMG is developing a comprehensive model of the teaching process (Schoenfeld, 1998, 2000; Schoenfeld et al., 2000; Sherin et al., 2000; Zimmerlin & Nelson, 2000). The work builds upon previous work that utilized a goal-driven architecture to examine tutor–student interactions (Schoenfeld et al., 1992). The model gives a detailed description of moment-to-moment interactions in terms of the teacher's goals and actions. Various factors are postulated to influence the ways in which teacher goals are developed. These factors include social context, subject matter knowledge, pedagogical knowledge, and beliefs. In conjunction with the on-going development of the TMG's model of teaching, this article centers on the role of beliefs in the model. Specifically, we attempt to establish that beliefs play a central role in the description of teacher practice. Specifically, we analyze the ways in which beliefs influence the formation and prioritization of goals.

As described by Schoenfeld (2000), the TMG uses empirical methods for parsing transcript records of classroom actions. For each lesson segment, we identify associated goals and trace their development through the interaction. Using fine-grained analysis of videotape and audiotape, a lesson is parsed into a sequence of episodes, each of which contains a coherent set of actions called "action sequences." We then infer elements of the teacher's decision-making processes, such as goals and "action plans" (which describe, at different levels of grain-size, what the teacher does to carry out her goals) that influence the moment-to-moment interactions.

The attribution of the goals and action plans is achieved through a process of competitive argumentation (see, e.g., Schoenfeld, Smith, & Arcavi, 1993; VanLehn & Brown, 1982). In addition, the researchers attempt to triangulate the data wherever possible. Additional sources of information include interviews with the teachers, "video club" (Frederiksen, Sipusic, Gamoran, & Wolfe, 1992) discussions of the lessons, and conversations with the teachers about the analysis.

Fig. 1 gives an abstracted graphical representation of a goal parsing produced during of analyses. The lines of transcript, which represents the dialogue and the actions of the teacher and students, are numbered and listed on the left. The goals are listed across the top of the figure and the vertical bars on the right-hand side show where particular goals were active. Dotted bars indicate that a particular goal was suspended during that portion of the interaction.

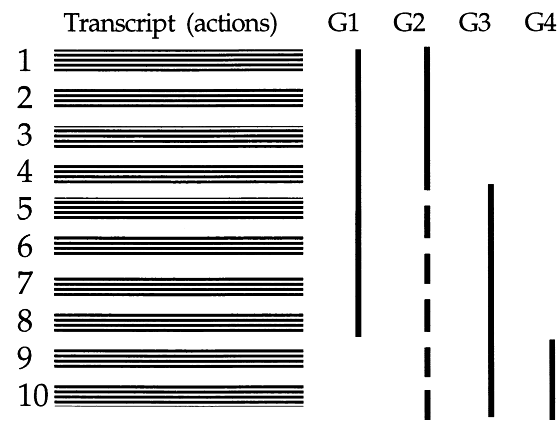


FIGURE 1. Graphical representation of a parsing.

4. CLAIMS, FRAMEWORK, TERMINOLOGY, AND METHODS

In this section we introduce our claims and describe the framework used to analyze the relationships among beliefs, goals, and practice. We discuss our specific use of the terms "belief" and "goal" and introduce the ideas of *goal shift* and *belief bundle*. We also introduce a representation for depicting the relationship between goal shifts and belief bundles in the context of teacher action. The section concludes with an introduction to the methods used in the study.

4.1. Claims

We make two central claims:

- Beliefs play an influential role in shaping the moment-to-moment practice of teaching.
- Beliefs are most likely to be apparent when a shift in the teacher's goals occurs.

4.2. Beliefs and Goals

For the purposes of this study, we define *beliefs* as personal philosophies (often implicitly held) consisting of conceptions, values and ideologies (Ernest, 1989). *Goals* are cognitive constructs that describe (at various levels of detail) what the teacher wants to accomplish.

We have found it helpful to organize beliefs into a hierarchy based on a structure proposed by Green (1971). Green described a quasi-logical organization scheme used to characterize beliefs. The hierarchical organization is based on how the person holds his or her beliefs, with some beliefs being fundamental and other beliefs being related to or derived from the fundamental beliefs. This structure orders beliefs based on how basic they are to the person's belief system. Beliefs fundamental to the person's belief system are given the top position in the representation. Beliefs that are ascribed or related to the top-level beliefs are located further down the hierarchy.

We distinguish between "attributed" and "professed" beliefs and goals. "Professed" refers to the beliefs or goals that teachers describe themselves as having. These are the beliefs and goals that the teacher identifies and articulates during the course of an interview or other discussion. These beliefs and goals may or may not be consistent with the behavior of the teacher that is observed during instructional interactions. "Attributed" beliefs and goals consistent with the teacher's practice are identified by the researchers. These beliefs may or may not correspond to what the teacher professes. To have as accurate a portrayal as possible, we use both attributed and professed beliefs and goals in this work.

Since we are talking about mental constructs, these attributed beliefs and goals are not necessarily the exact ones that teachers hold or would articulate, but they are the constructs that are consistent with and enable us to explain the teacher's actions. When we state that the teacher has such a belief (or goal), we mean that *the teacher is behaving in a manner consistent with having such a belief (or goal)*. The attributed goals may be long term or very broad in their scope (overarching), or short term and immediate (local). In addition, they can be either pre-existing or emerge during the classroom interaction.

4.3. Goal Shifts

Goals can account for particular actions in teacher practice. A change in these goals can make the beliefs apparent and enable us to examine their influence on the formulation of a new goal. Several beliefs are likely to be influencing the teacher's decision-making and goal-formation processes at the same time. We may or may not have access to all the beliefs at a particular goal shift, but we contend that a significant number of the beliefs relevant to the goal shift are apparent.

4.4. Belief Bundles

The particular beliefs that are apparent at these shifts are connected to one another in what we call a *belief bundle*. The belief bundle is a collection of beliefs that influence the formulation of a goal. The belief bundle serves as a set of constraints on the formulation of the new goal and has two characteristics. First, it connects particular beliefs from various aspects of the teacher's entire belief system (beliefs about learning, beliefs about teaching, etc.). Second, the activation level of certain beliefs in the bundle can help explain the emergence and formulation of the teacher's goals.

Fig. 2 is a representation of the transcript (which embodies the teacher's actions), goals, and belief bundles. The example in Fig. 2 involves a portion of transcript (on the left-hand side), two goal shifts, and two associated belief bundles (on the right-hand side). In this example, the first bundle is composed of beliefs b1, b2, b3. The beliefs that make up the bundle are called *component beliefs*. These beliefs may or may not appear together in the formation of other belief bundles. For example, beliefs b2 and b3 may appear in another belief bundle that does not contain b1. In addition, the beliefs maintain their respective positions in the teacher's overall belief system (as teaching beliefs, learning beliefs, etc.). A bundle is a particular manifestation of certain beliefs at a particular time.

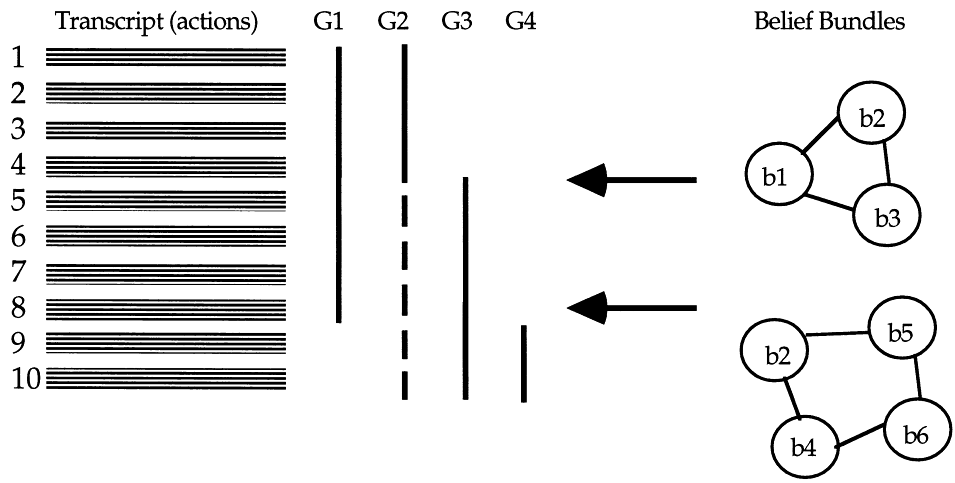


FIGURE 2. Graphical representation of a parsing with belief bundles.

A teacher may hold different beliefs about mathematics, teaching, and learning, some of which may form a belief bundle. For example, the belief that formulas and equations are the essence of mathematics (b1) might be associated (in the process of goal-formation) with the belief that students learn by listening to the teacher (b2) and the belief that the teacher needs to present a clear sample problem to orient student learning (b3). In Fig. 2, these three beliefs influence the formation of the new goal (G3) at line 5 of the transcript. The second belief bundle is associated with the goal shift at line 9. This bundle may or may not contain some of the same component beliefs as the first bundle. In the goal parsing example in Fig. 2, the two belief bundles that became apparent have a common component belief (b2), the belief that students learn by listening to the teacher.

We contend that a bundle can consist of a number of component beliefs. However, there is not necessarily a pre-determined number of beliefs that defines a bundle. The beliefs identified as being in the bundle are those beliefs that appear to be particularly influential on the formation of the new goal. This does not imply that these are the only beliefs held by the teacher. It only implies that they are the beliefs that are the most salient and are exerting the strongest influence on the formulation of the new goal at that particular moment in the interaction.

4.5. Methods

We examine the belief–goal connection in the practice of two Algebra I teachers. The research is informed by two previous studies of these teachers' beliefs and practices (Aguirre, 1995; Speer, 1996). Both studies employed qualitative methods to capture the relationship between beliefs and practice. These methods included fine-grained analysis of videotape, teacher interviews, classroom observations, and field notes. These previous studies provided the information about the teachers' beliefs about mathematics, teaching, and learning used in the present study.

The present study employs methods similar to those used by the TMG in research previously described. We identified episodes of videotape where the teacher formulated a new goal in reaction to an unanticipated student response or question. These episodes were selected based on our conjecture that one place we might see the influence of beliefs on practice would be at a shift in the teacher's goals. The parsing of action sequences revealed certain shifts in goal formation. These shifts corresponded to particular teacher actions and we hypothesized that they were also related to the teacher's beliefs. Details of how these methods were used will be explained below in the context of the classroom examples.

5. CASES

Each case example described below begins with a brief outline of the history and background of the teacher, a description of their school and classroom setting, and a summary of their beliefs derived from classroom observations and interviews. Next, the classroom episode is presented and analyzed. Then, we present the transcript of the episode, followed by a description of the goals and an explanation of the role of the belief bundle in the shift in goals.

5.1. Case Study #1: Ms. Perry

The first example is drawn from a series of observations of an eighth-grade Algebra 1 class in Spring 1995. The teacher, Ms. Perry, is the chairperson of the mathematics department in a large, K-12 independent school. She has taught many different courses in her nearly 20 years of teaching experience and was teaching problem solving and first-year algebra courses during this semester. She has also been making many changes to her teaching practice in response to mathematics reform documents such as the NCTM Curriculum and Instruction Standards (1989). As part of these changes, in addition to participating in whole-class discussions, Ms. Perry's students spend time working in small groups on problems. She has created a classroom environment in which students appear to be comfortable and to work productively.

Teacher beliefs. Teaching, learning, and the nature of mathematics are categories of beliefs commonly identified as having an influence on a teacher's practice (Thompson, 1992). Sometimes beliefs about students' feelings toward school or particular subjects are placed within the student or learning belief categories. However, in examining and analyzing Ms. Perry's beliefs it became apparent that her concerns for various affective issues related to her students were not just a part of her beliefs about teaching and learning; they comprised a distinct section of her belief system. In light of this, the description of her beliefs has been organized into four sections: beliefs about affective issues, the nature of mathematics, learning, and teaching.

The following section provides a summary of Ms. Perry's beliefs based on interviews with her. A more detailed description of this particular teacher's beliefs can be found in Speer (1996). Fig. 3 is a representation of the major parts of Ms. Perry's belief system.

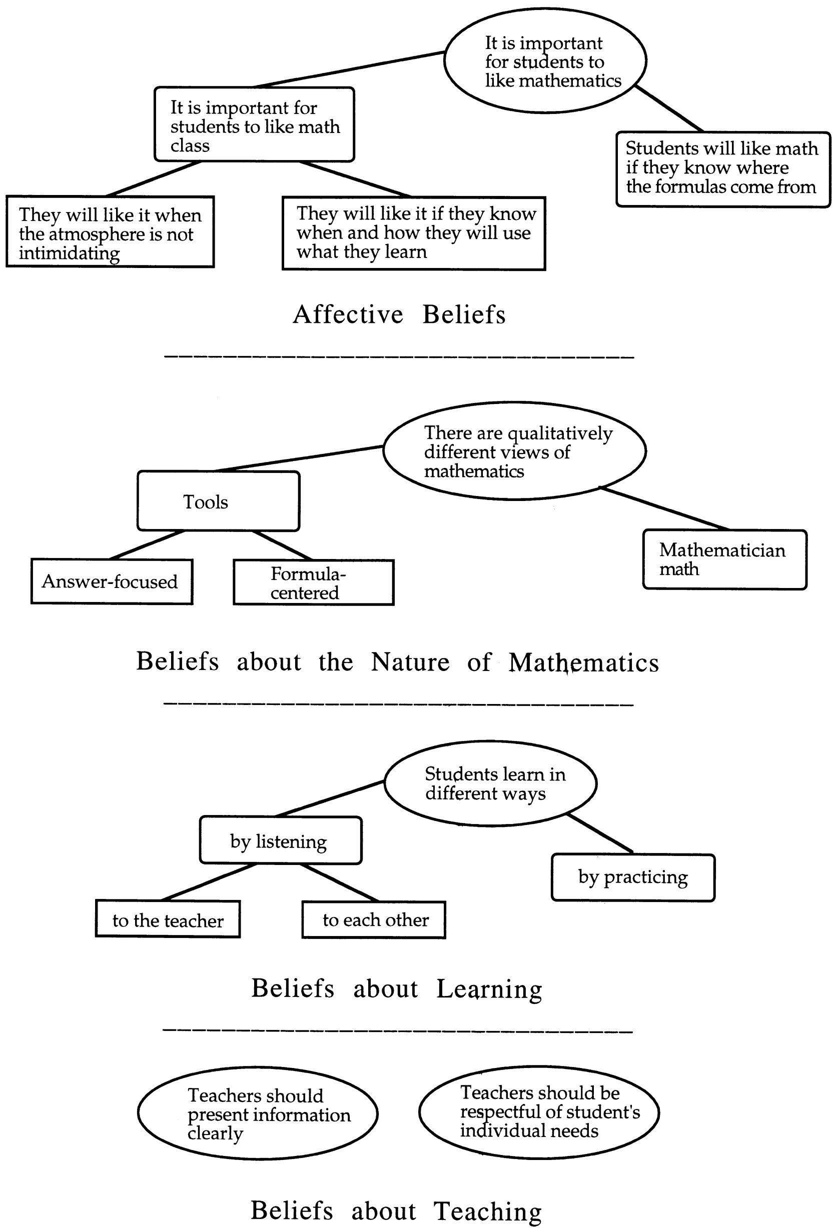


FIGURE 3. Ms. Perry's beliefs.

Beliefs about affective issues. Ms. Perry entered mathematics teaching because she saw in students a strong dislike and fear of mathematics. Her experiences as a mathematics student led her to believe that aspects of the atmosphere in the math classes might be contributing to the negative feelings the students had about mathematics and mathematics classes. Her primary affective belief is that it is important for students to like math. She "had this real suspicion that kids didn't like math" and that students viewed math teachers as "real nerds" who only care and think about math issues and spend all their time doing math problems or playing with calculators. She "was really shocked that they would think that math could be that bad" and she wanted to do something to change the students' attitudes.

Ms. Perry believes that, in addition to not liking the subject of mathematics, students often do not like their math classes. She thinks that part of what contributes to students' dislike of mathematics is "the attitude in the class." She described this attitude as the intimidating nature of the interaction between the teacher and the students. She believes that many math teachers think "fear was the component that gave them control over the class." She remembers having teachers who "carefully worked on their physical presence to make sure that you were sufficiently intimidated to sit up and behave" and she feels "that's not the way to teach math at all."

Ms. Perry also believes that if students don't know if or when they will ever use what they are learning, they will not like math class. It is very important for her students to feel that what they are learning will be of use to them since that will make them enjoy learning. She wants her students to come away from her math class understanding that what they have learned is information that "they're going to apply in just about any subject they go into." And in a more general sense, that what they are learning in math class "is something that you can really use."

Ms. Perry also believes that it is important to teach by showing students where the various formulas and equations are derived from. Part of why students don't like mathematics is that teachers often present formulas without any explanation about where they come from—the "mystery of mathematics." She thinks that it is very important to show students how particular formulas were arrived at so they have some sense of how the new information connects to what they already know. This connection helps them understand the material (a teaching/learning belief) but she also thinks it plays a crucial role in how students feel about mathematics.

Beliefs about the nature of mathematics. Ms. Perry believes that there are qualitatively different kinds of mathematics. To her, mathematics can either be seen as a collection of tools or it can be seen as a theoretical study of patterns. She is familiar with these alternative views of mathematics and she has made a deliberate choice to convey the view of math as a collection of tools to her students. Although she describes these two alternative view of mathematics, she says that personally she views math as a collection of tools.

There are two aspects to her view of mathematics as a collection of tools. First, she sees mathematics as composed of an inventory of formulas—it is not composed of the ideas or relationships associated with them. She believes that her students "need a set of calculations" to use in their future occupations. This collection of tools is comprised of the formulas, equations, and specific techniques that allow people to find the numerical answers to problems. She is convinced that nearly all of her students will go on to study and work in areas that require them to have an understanding of mathematics as a set of tools used to solve various problems.

Second, she believes that being able to get the answer to a problem is what is important. Her view of the nature of mathematics makes the practice of mathematics an answer-focused activity. She does not believe that the social atmosphere of the classroom should focus on getting the *correct* answer (because of her various affective beliefs), but she does believe that the goal of learning mathematics is to be able to find answers accurately.

Another kind of mathematical understanding comes from the study of mathematics as a pure science. This kind of understanding includes the exploration and analysis of patterns and relationships. This alternative view of the nature of mathematics has to do with the kind of mathematics that mathematicians need to know. She believes that the theoretical understanding of mathematics does not contribute to students' ability to use the math tools and is therefore important only for people who will be pursuing pure math. She thinks that people who end up going into pure mathematics understand things differently than other people. When asked about the students who do go on to study mathematics, she said their view and understanding of mathematics "will become a study of patterns or a making some kind of sense out of chaos." She thinks that the students who go on to study mathematics will take the collection of tools that they have and, over time, transform their work in math into more of an investigative and theoretical endeavor. She believes that the kind of understanding necessary to be a mathematician is something that certain people are born with—"they've had it all along. I don't think we've [teachers] put it in there."

Beliefs about learning. Ms. Perry has two basic beliefs about learning: people learn by listening to information and people learn by practicing solving problems. In describing her own experience in learning mathematics, she remembers an algebra teacher whom she felt was effective. She said "... he was very clear and I think I got a good set of practice problems from that class. ... I think it was probably a pretty good explanation." When asked if she thought she learned the math by listening to the teacher's explanations or by trying the problems herself she replied "Both. Definitely both." She feels this is true of how her students learn as well.

She believes that it is important for students to either hear an explanation of a new skill from her, or for them to read about it in their textbook or both before they attempt to use the new skill to work any problems. It is during this time that she provides them with information about where the new equation or formula comes from and how they will be using the skill in various problems that they solve now and in the future.

There is one particularly important consequence of her belief in learning by listening. Because of the strength of this belief, she focuses only on the benefits to the students who are doing the listening and not on the benefit to the student doing the speaking. As a result, much of her teaching practice is focused on the characteristics of what is being said in the classroom and not on individual student's thinking about the mathematics.

Her second primary belief about learning is that people learn by doing problems. She thinks that students get the chance to see which ways of solving problems are most effective or efficient. Students learn a lot by trying to solve problems and by seeing which tools were useful to them in the various situations. She thinks that having students do problems and then discuss them as a whole group provides opportunities for the students to see what they did that worked and what they did that didn't work as well.

Beliefs about teaching. Ms. Perry has two primary beliefs about teaching. She believes that as a teacher it is important to be clear and to be respectful of one's students. She believes that these things are not just part of how she wants the atmosphere to be in her classroom but that they are crucial to the success of her students' learning.

She believes that it is important for the students to experience a very straightforward, direct, and clear presentation of the material. It is important for students to share and discuss the methods that did not work but she believes that when a topic is being introduced it is important for students' incorrect solutions to be intercepted. This is to ensure that the wrong ideas are not floating around for other students to hear, at a formative stage when the students are not very familiar with the correct methods. Since she believes that students learn by listening to explanations and problem solutions she feels that it is important for them to have the opportunity to hear correct ones the majority of the time.

She also feels it is important for her and for students to treat each other respectfully. She thinks that it is very important to create and maintain a pleasant classroom environment. As discussed previously, she believes that students "learn best when they're not afraid" of the teacher or afraid of being embarrassed or made to feel insecure about their math ability. Although this belief is related to some of her affective beliefs, it is also a primary aspect of her beliefs about teaching.

Episode background. The episode discussed here occurred during an introductory discussion of the point–slope form of a linear equation. Prior to this lesson, the students spent several weeks studying the slope–intercept and the general linear forms of the equation of a straight line. They had read about the derivation of the point–slope form in their textbooks for homework the night before this class, but had not been asked to work through any problems.

Ms. Perry had created a worksheet for the students to use during this class. The worksheet guided students through a set of increasingly complex situations where they needed to write equations in point–slope form and to demonstrate the equivalence of the various forms of an equation. The first few problems were done by the class as part of a whole-group discussion and the rest were assigned later for homework.

The first problem asked the students to recall the point–slope form from their reading:

> Write the formula down for an equation of the line in point–slope form. Use (x, y) and $(x1, y1)$ for your points. Use m for your slope.

Ms. Perry had already discussed this problem with the students and had presented a derivation of the formula $y - y1 = m(x - x1)$. The following steps in the derivation were on the board:

$$(x,y) \quad (x1,y1) \leftarrow \text{ known} \quad \text{slope } m \leftarrow \text{known}$$

$$m = \frac{(y-y1)}{(x-x1)}$$

$$\frac{m}{1} = \frac{(y-y1)}{(x-x1)}$$

$$y - y1 = m(x = x1)$$

The transcript excerpt presented below is the class discussion of problem #2:

> Write an equation of the line in point slope form for a line that passes through the point (0, 12) and has a slope of 2/3.

After reading the problem and writing the relevant values on the board, Ms. Perry asked for a student volunteer to present the solution to the problem. See the left-hand side of Fig. 4 for the transcript.

Episode parsing. Four goals can be attributed to the teacher during this interaction. The right-hand side of Fig. 4 gives the complete goal parsing representation. One of Ms. Perry's major goals is to ensure that the solution to problem #2 is presented clearly to the students (G1). Associated with this goal is a second goal: to have a student volunteer do the presentation of the solution (G2). These two goals are active in lines 1–6 as she is reading the problem, writing the relevant information on the board, and soliciting a student volunteer.

In line 7, the student appears to have some difficulty producing the solution. Ms. Perry was expecting the student to plug the values for $x1$, $y1$ and the slope into the formula to obtain $y - 12 = 2/3(x - 0)$. She anticipated that the student would begin with "$y - 12$," but instead the student starts with the value of the slope. Ms. Perry, suspecting that the student might merely be starting with the right-hand side of the formula instead of the left, writes 2/3 on the board. If this was indeed what the student intended, she would have continued by telling Ms. Perry to multiply by $(x - 0)$. However, in line 7 the student says "is equal to ... ," and Ms. Perry realizes that the student is not presenting the intended solution by following the formula.

As best as we can tell from the video record, the student is attempting to follow the steps that Ms. Perry had presented to the class when she derived the formula. The student seemed to be starting to produce $2/3 = (y - 12)/(x - 0)$, which mimics one of the equations in the derivation that was still on the board. This is a reasonable reaction given Ms. Perry's earlier presentation (especially since the slope in the problem is a fraction)—but it is not the response she expected.

The student's response prompted Ms. Perry to point to $y - y1 = m(x = x1)$ on the board. Her intention was to direct the student to the equation, and hence the correct solution path (G3). During this time her goals of having a solution presented clearly and of having the student volunteer do the presentation (G1 and G2) remain active but would be in conflict if the student was not re-directed to the correct equation. The interaction continues until lines 11 and 12, which together trigger a shift in goals.

At this point, the student still has not begun the correct presentation of the solution. However, this time, Ms. Perry does not respond by pointing to the equation. Instead, Ms. Perry decides to lead the student through the steps to the correct solution and the goal of having the student present the solution on her own (G2) has now been suspended. Ms. Perry has not abandoned this goal as can be seen in the fact that she continues to give the student opportunities to take over the solution presentation in subsequent lines. However, the goal of getting the student on the correct path becomes a very high priority new goal, and guiding the student through the solution using intensive scaffolding (G4), emerges. Ms. Perry guides the student

T So look at #2, x1, y1 is the point we know [writes (x1, y1) and (0, 12)], it's 0,12. The slope we know is 2/3rds [writes m = 2/3]. Brave volunteer [two students raise hands], tell me what to write for an equation in general point-slope form. S, go ahead.

S Okay, do 2/3rds ...

T Okay [writes 2/3]

S ... is equal to, um ...

T Let's do this form in the box [points to formula]

S Yeah.

T We're going to use this one. Okay the 2/3rds is ...

S Oh yeah, one, oh, okay. So, ... three.... um, times, [CE doesn't write anything] is the, should the three be to the x's?

T Can you give me the difference of the y's? [points to formula] Can you tell me what...

S Oh the y's?

T .... numbers you'd put in for the difference of the y's, just to start it out? We don't know y [writes y].

S But, so y and then 12?

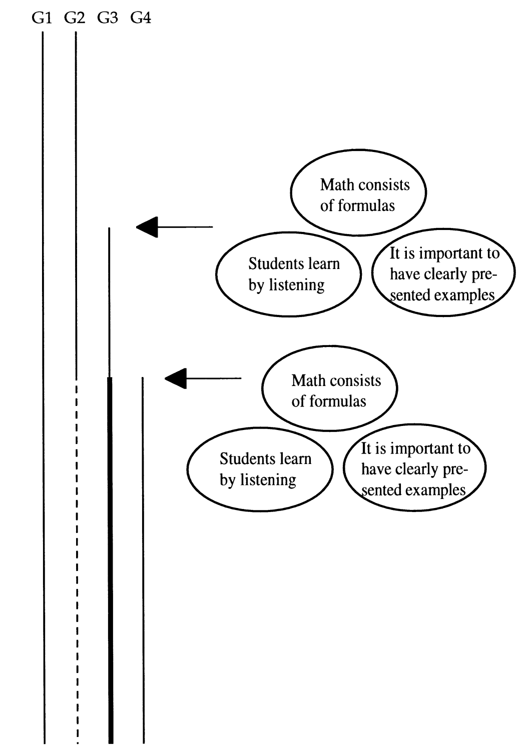
T But we do know 12 [writes y - 12]. Good. And we know that's equal, to the slope ...

S And then how's that, after you do that, don't you do that or no?

T ... times the difference of the x's. Can you tell me how to write the difference of the x's?

S Yeah, you do x, um, x minus zero.

T x minus zero! [writes x - 0].



Goal legend corresponding to the list of goals on the right-hand side above:

G1: Have example presented clearly.
G2: Have student produce example/solution.
G3: Get student back on track/back to equation.
G4: Lead student through solution.

FIGURE 4. Transcript, goal parsing, and belief bundles for Case #1.

through the steps of putting the correct values in the equation and by line 25 the correct answer is on the board.

Role of the belief bundle. The decisions Ms. Perry made were influenced by how adequately the current situation advanced her goals. When something happened that she saw as being or having the potential to become counterproductive, she intervened. She permitted, but only to a certain extent, the interactions to continue that were not in direct service of her goals. The interventions took the form of redirecting the flow of the discussion and were minor at first, but then became more extensive.

In this particular interaction, there were two goal shifts. At each shift a belief bundle became apparent. As it happens the component beliefs of the bundle were the same for both goals shifts. Each belief bundle was comprised of three: one belief about learning, one about teaching, and one about the nature of mathematics. These bundles influenced the decisions she made.

The belief bundle is shown to the right of the goal traces in Fig. 4. In this belief bundle, the pertinent belief about the nature of mathematics is "math is a set of tools or formulas." The learning belief is "students learn things by listening to other people talk through a solution to a problem." The relevant teaching belief is "it is important for students to have examples and information presented to them clearly."

The first emergent goal was formed in response to the student's solution attempt in line 7. The student appeared to be having difficulty producing the correct solution to the problem. This challenged the teacher's goal of having the solution presented clearly. In an attempt to get the interaction back on track, she pointed to the equation and told the student to use a particular form of the equation. Since the student was not demonstrating the correct use of the formula, this bit of interaction conflicted with her belief that the use of formulas and equations is extremely important in mathematics. The line of reasoning that the student was starting to present was not explored or discussed; since it did not include the proper equation, it was deemed inappropriate. At the same time, the teacher's belief that it is important for students to have a clear example when they are learning a new topic was challenged. The student volunteer was supposed to be the source of the presentation of the solution and her apparent confusion was not yielding such a presentation. These challenges to the beliefs in the belief bundle influenced the formation of the new goal of guiding the student to use the correct equation.

The second emergent goal was formed in response to what the student says in lines 11 and 12. In spite of the teacher's effort to redirect the student to the correct equation, the student still seems lost. At this point, beliefs from the bundle are challenged again. In the previous interaction, Ms. Perry's learning belief was not challenged as extensively since the confusion seemed minor. When the teacher re-directed her to the equation, the student responded in a way that indicated that she understood. In the second instance, however, the student still had not started to present the solution to the problem, and from the teacher's perspective this ran the risk of confusing the other students. Since this situation still challenged the beliefs in the bundle, a more dramatic shift occurred. A higher activation of Ms. Perry's belief that it is important to have clearly presented examples caused her to focus more on her goal of getting the student to use the correct equation. The prominence of this goal is represented by the bold goal trace of G3 in Fig. 4. This also meant that Ms. Perry could no longer pursue the goal of allowing the student to make the presentation on her own. In addition, a new goal emerged. Now, Ms. Perry guided the student through the solution more directly, while still allowing the student to provide pieces of information that contributed to the solution.

The above example illustrates the belief-goal connection of a teacher during whole-class interaction. To provide additional evidence that beliefs become apparent when goals shift during teacher–student interactions, we will now focus on a different classroom activity structure: student group work.

5.2. Case Study #2: Mr. Martin

In 1993, the Functions Group at the University of California, Berkeley began working with Mr. John Martin, a mathematics teacher at a local urban high school that has an ethnically, linguistically and socio-economically diverse student population. The Functions Group had developed a 6-week research-based curriculum unit on linear functions that incorporated small-group activities and computer technology (Lobato, Gamoran, & Magidson, 1993). Members of the Functions Group worked with Mr. Martin and another teacher who implemented the curriculum in the Spring semesters of 1993, 1994, and 1995.

Mr. Martin is a veteran public school mathematics teacher with 11 years of experience. He wanted to participate in this research project because he was interested in improving his teaching practices. One of the areas he wanted to focus on was promoting collaborative learning through small groups. He felt the curriculum and instructional support of the project would give him the structure to make the "big break" from his teacher-centered practices. The curriculum unit on linear functions implicitly promoted a collaborative agenda by incorporating many small group activities throughout.

As previously discussed, it is necessary to understand Mr. Martin's beliefs about mathematics, teaching and learning in order to better understand his teaching actions. In this particular case, the teacher is in flux, operating from two distinct perspectives that were built from his beliefs and influenced his practice. Following the discussion of his beliefs, an example of an episode of small-group interaction that occurred in the second year of the project will be given. This example illustrates the influential role particular beliefs play in goal formulation and teacher action.

Teacher beliefs. In a previous study (Aguirre, 1995), the connection between Mr. Martin's beliefs and practice was articulated by focusing on the teacher's professed beliefs and his actions during small-group activities. Although the analysis did not address goal formulation specifically, it demonstrated that his behavior during small group interaction was informed by two distinct perspectives built from his professed beliefs. One perspective encompassed his traditional or "standard" beliefs about mathematics, teaching and learning, and another consisted of newly formed beliefs that reflected a more collaborative view of teaching and learning.

The teacher's dual belief system consists of parallel perspectives on mathematics, teaching and learning built from his beliefs. Figs. 5 and 6 provide a representation of this two-part system. His standard beliefs were well developed and

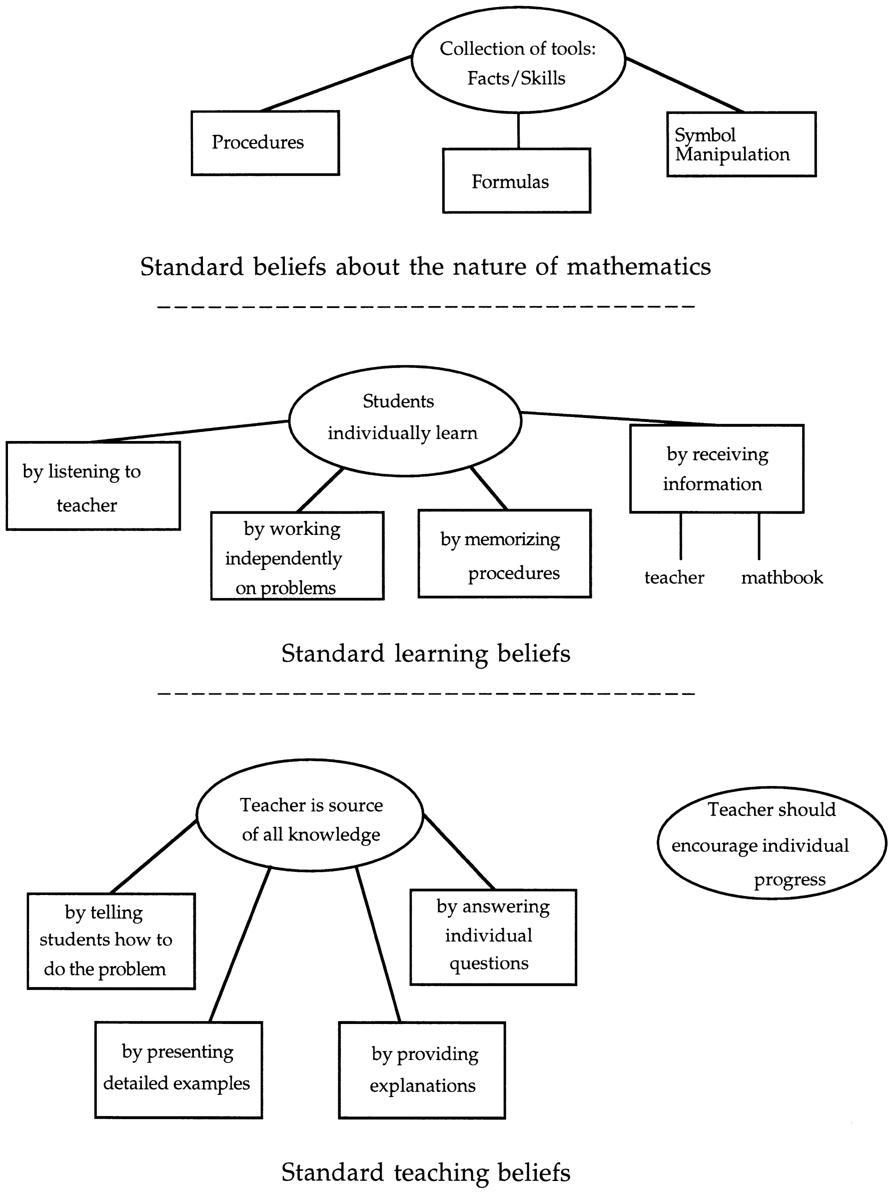


FIGURE 5. Mr. Martin's standard beliefs.

steeped in 11 years of teaching mathematics. The collaborative beliefs were emerging, and somewhat fragile, as they continued to develop over the course of the unit's implementation. The architecture of these beliefs and how they impact behavior is discussed elsewhere (Aguirre, 1995). However, it is important for the present discussion to elaborate on some of the most salient beliefs in the teacher's dual system.

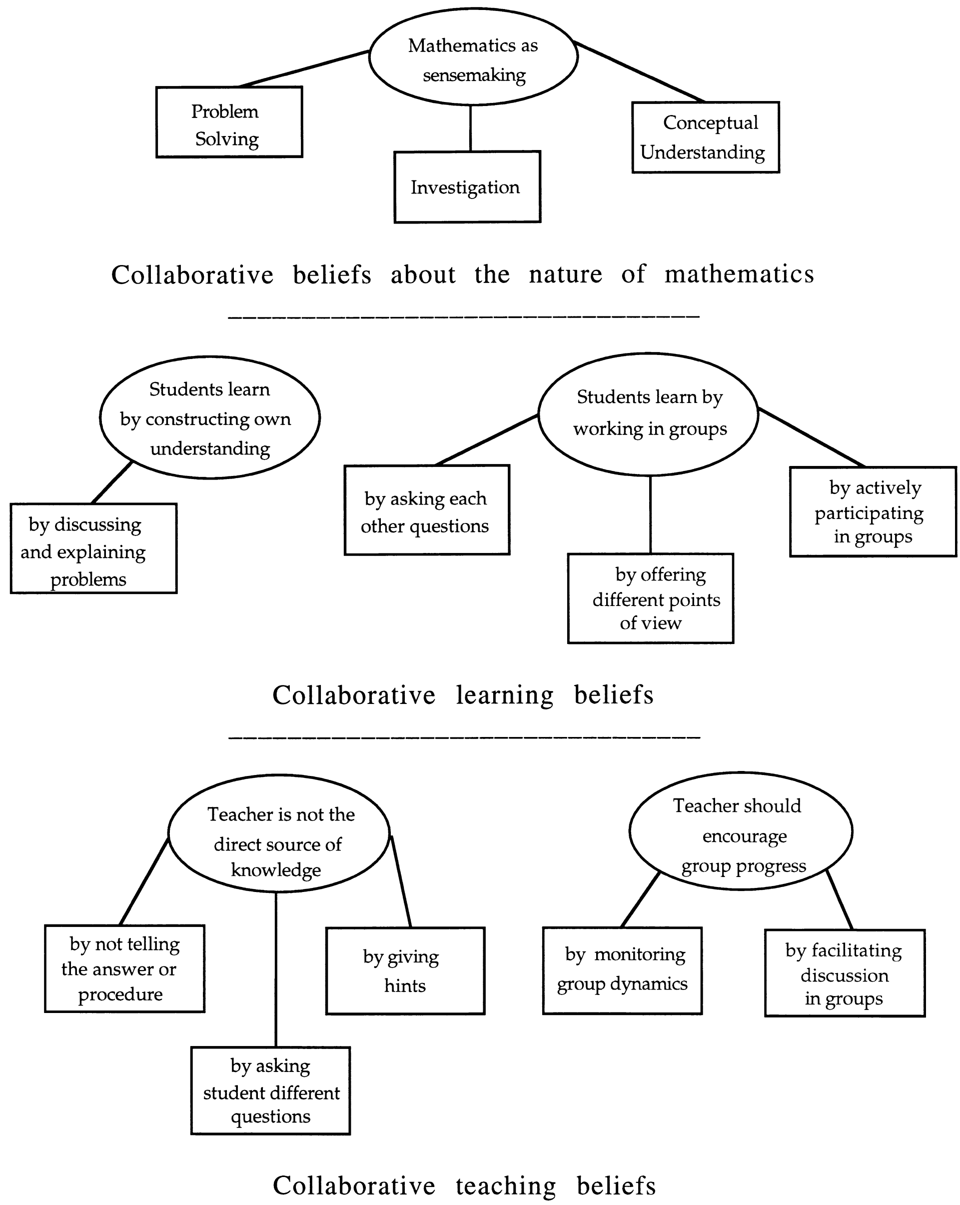


FIGURE 6. Mr. Martin's collaborative beliefs.

Mathematics. Mr. Martin believes mathematics is a set of useful, but often unrelated facts, rules and skills, with emphasis placed on procedural understanding. The texts Mr. Martin used in his algebra classes presented and reinforced a similar view of the subject matter.

In contrast, the curriculum unit on linear functions reflected a different perspective of mathematics that emphasized conceptual understanding, problem-solving, and investigation. This approach to the development of mathematical concepts (such as slope) was a stark contrast to the way in which Mr. Martin usually taught the subject. For example, Mr. Martin traditionally presented the slope formula on the second day of the unit on slope, while the new curriculum unit addressed the slope formula in week four. Delayed placement of the formal definition was done deliberately to allow time for students to develop a strong conceptual understanding of slope. Mr. Martin indicated that the traditional approach he had used to teach mathematics was limited. He admitted that it was unclear whether his students understood and/or could demonstrate the concept of slope beyond symbolic manipulation and solving equations.

Mr. Martin's intent to foster collaboration hinged upon the curriculum unit. Although he indicated he was more comfortable with the unit and its mathematical approach in the second year, he did not think students could "develop their own understanding" and learn to work collaboratively in small groups unless the curriculum structured those interactions. And, for six weeks, his traditional view about mathematics was challenged by an alternative view supported by the use of the curriculum unit. This dual view about mathematics held by the teacher is connected directly to his beliefs about learning and teaching.

Standard theories on learning and teaching. Mr. Martin's standard beliefs about learning and teaching are represented in Fig. 5. Given the emphasis on procedural understanding in the traditional algebra curriculum, Mr. Martin's beliefs about learning focused on the importance of students being able to manipulate equations. Student learning was characterized by symbol manipulation and procedural understanding of solving equations. Mr. Martin reported learning as "passive," as the students sat individually at their desks and listened to the teacher explain how to solve a problem.

Mr. Martin described his teaching outside of the collaborative curriculum as "lecture style," where he sat in front of the class and orchestrated a traditional recitation discussion (Stodolsky, 1988) by presenting the material, calling on students to answer questions, explaining a few problems for guided practice, and giving students time to do individualized seatwork on the problems. In addition, he explained that during his standard teaching he was the source of all knowledge. He believed that if he did not explain the lesson or topic thoroughly to the students, the students would not learn it. He identified his ability to explain things and break points down to their essential components as one of his teaching strengths.

Another salient teaching belief is the importance of encouraging students to make "progress." For instance, if the students were struggling and on the verge of giving up, Mr. Martin said he directly intervened by telling them explicitly what to do in order to "aim them back towards the trail" and "move forward." The importance of encouraging student progress, a belief that spanned both the standard and collaborative teaching theory, informed an over-arching goal of wanting the students to make progress during class on the activity. It is also important to note that in the standard model, encouraging progress focused on each individual student making progress on a particular problem. Thus, the teacher's particular view of mathematics and beliefs about teaching and learning are contributing factors to how the teacher encourages progress.

In summary, this standard model promotes independent learning in isolation as well as didactic and unidirectional teaching. In the classroom environment created by the teacher, students working in groups was an infrequent occurrence and implicitly discouraged. However, it must be made clear that Mr. Martin recognized the limitations of his standard teaching practice and the type of learning that had traditionally resulted from it. Therefore, it is also important to discuss his collaborative theories of learning and teaching that represent this teacher's transition toward working with small groups.

Collaborative theories on learning and teaching. Mr. Martin's collaborative beliefs are represented in Fig. 6. Mr. Martin's professed beliefs about collaboration were most salient when it came to discussing the impact of fostering collaboration on learning. He asserted strongly that working collaboratively in groups helped students to construct their own knowledge and understanding. Discussing with peers, explaining to each other, and offering a point of view were all examples that facilitated student learning. The teacher clearly articulated some of the learning benefits for students working in groups in comparison to the more passive learning style supported by his standard teaching practices.

However, there is one important element of collaboration documented in the literature that was missing in this learning theory. No emphasis was placed on how the students were to make sense of the subject matter (Barnes & Todd, 1977; Schoenfeld, 1994; Yackel, Cobb, & Wood, 1991). Mr. Martin's learning theory included students asking questions and discussing the mathematics with each other—but the kinds of questions or the particular ways students should discuss the mathematics that reflect higher order thinking and promote deeper conceptual understanding were not identified. Nothing was brought to bear on how the students were actually supposed to engage the mathematics using each other and the teacher as resources.

Mr. Martin's model of collaborative teaching, to some degree, conflicted with some of his standard teaching practices. For instance, he recognized that telling students the answer or how to do a problem constricted student engagement with the mathematics. One of Mr. Martin's evolving collaborative teaching beliefs was the importance of not explicitly telling the student the answer or how to do the problem. In lieu of directly telling students, he developed some alternative teaching beliefs that emphasized the importance of giving hints and asking students questions about their solutions. He changed his notion of the teacher as the source of all knowledge to teacher as the monitor—someone who did not directly intervene, but encouraged students to work together, and continually checked their mathematical progress.

The change in teacher role reflected a different instantiation of Mr. Martin's belief about encouraging students to make progress. This belief remained strong in his collaborative model and took on an enhanced form that included encouraging the progress of student groups. The tenuous state of his beliefs about collaboration was evident in his practice as he tried to find a balance between what had worked in the past and what he thought he needed to change.

Situated alongside his traditional beliefs, the newly developed and somewhat unstable collaborative beliefs could appear to be in direct competition with each other. However, one could argue an alternative view that the traditional beliefs are being suspended while the collaborative beliefs evolve in the context of implementing a new teaching practice, working with groups. Therefore, the beliefs are not in direct competition, but in different states of activation depending on the instructional context.

In addition, the fragile state of Mr. Martin's collaborative teaching beliefs was supported by a limited repertoire of routines and strategies. Unlike his highly refined routines and strategies that are embedded in his traditional beliefs, his pedagogical skills and routines for fostering collaboration were just developing. For example, one of Mr. Martin's collaborative teaching beliefs relates to facilitating collaborative dialogue. Mr. Martin indicated that he was unsure how to actually promote collaborative dialogue within the groups outside of general group encouragement such as, "That's for you guys to figure out." Although his teaching beliefs support collaborative groups, his limited understanding and underdeveloped pedagogical strategies and routines to help students effectively participate in groups were indicative of the tenuous state of his collaborative teaching theory.

In summary, the dual belief system presented in this section identifies specific teacher beliefs about mathematics, learning and teaching. Mr. Martin holds strong, well-connected theories of teaching and learning linked to his view of mathematics while using a traditional curriculum. It is unclear whether his view of mathematics changed with the implementation of the collaboration curriculum. However, his beliefs about the collaborative curriculum, however tenuous, offered a catalyst to promote development toward a more collaborative teaching repertoire.

Throughout the implementation of the curriculum unit, Mr. Martin's interaction in small groups reflected his collaborative beliefs about teaching and learning. As discussed in Aguirre (1995), this teacher had predictable responses to student questions that reflected his collaborative teaching and learning beliefs. Mr. Martin's interactions with small groups could be characterized primarily as monitoring with little intervention. If he did intervene, the character of his intervention was markedly different than his standard practice of telling the students how to do the problem. He would ask questions and give hints rather than explicitly tell students how to do the problem.

What makes the following example interesting is that, in response to a student's question, the teacher exhibits behavior characteristic of his standard teaching practices. The retreat to his standard teaching practice exemplified in the forthcoming episode suggests an interesting place to examine the role that beliefs play in the formulation of new goals.

Episode background. The example is presented as follows: First, the nature and background of the activity are described. Next, the transcript of the interaction is presented, followed by an analysis using our goal parsing methods. An explanation of the goal shifts that occurred in the interaction is given. Then, the nature of the particular beliefs comprising the belief bundle and their role in the formulation of new goals at these particular instances is discussed.

In this episode, the activity focused on a problem that integrated many of the concepts the students had been exploring over the six-week unit, such as slope, y-intercept, and the use of multiple representations (tables, graphs, equations). The problem stated that the students were working for a marketing department of a video-game company called "Pretendo." The goal of the activity was for the students to figure out the minimum number of games a customer would need to purchase in order for the Pretendo gaming system to be cheaper than the system sold by its competitor, Sega-Genesis. The problem outlined the cost per game and the cost per system of each company's product.

> Pretendo's total cost was US$220 (US$180 for system and US$40 per game). Sega-Genesis' total cost was US$170 (US$120 for the system and US$50 per game).

The students were asked to present their findings as if they were giving a presentation to the company's executives. In addition, the problem provided an explicit hint: "Think about how you can use what you've learned in this unit: Tables, Graphs, and Equations." Although each student would produce his or her own problem solution, the students were initially instructed to work on the problem in their small groups and finish up their solution as homework.

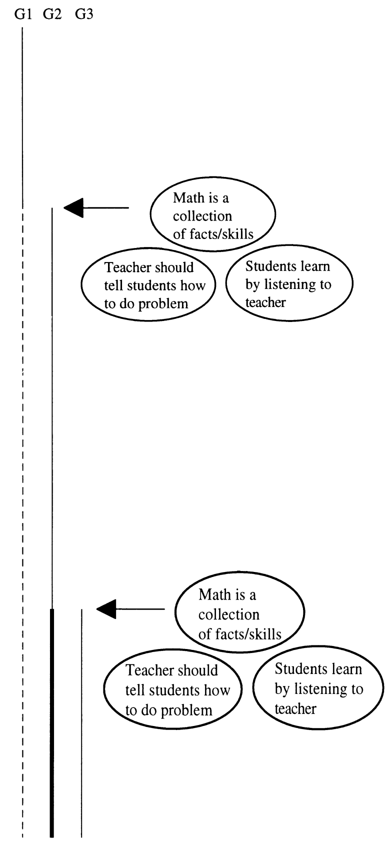
In this particular interaction, the students had been working on the problem for over 20 minutes. There were approximately ten minutes left in the period. There were nine large tables in the classroom with up to four students sitting at each table. The transcript (see the left-hand side of Fig. 7) begins as the teacher approaches one of the tables with three students working on the problem.

Episode parsing. Three goals can be attributed to this teacher in the episode parsing (see the right-hand side of Fig. 7). The first goal (G1) is a high-level collaborative goal of wanting the students to work together. This collaborative goal is active as the interaction begins. J raises her hand to get the teacher's attention. In line 1, J clarifies with the teacher whether her graph represents his previous suggestion[1] of including the cost of each company's system to the number of games. The teacher responds by indicating that she only needs to consider the cost of the system once (line 2). Apparently, her graph adds the cost of the system to the cost of each game. The teacher's overarching goal of progress for this student has not been breached.[2] Although her graph was incorrect for the purposes of the solution, his verbal exchange indicates that she is progressing. He responds to her question and checks for some acknowledgment of his response, and the student confirms she understands (line 5). Then, one of the other students at the table immediately asks him another question (line 6). The limited interaction with J (9 seconds) does not negate the existence of the high level collaborative goal of wanting the students to work together. The students are all working on the problem at the table. Therefore, we can infer that his goal of having the students work together collaboratively remains intact.

In line 6, D asks a question about including additional information with her solution. The teacher pauses and takes about 7 seconds to examine her work. The student has drawn a graph that represents the cost per game of each company's product. Her graph is mathematically correct, but not complete because it does not take into account the cost of each company's gaming system. It starts with the cost being US$0.00 for zero games, US$50 and US$40 for one Sega and Pretendo game, respectively, US$100 and US$80 for two games, etc. Since the student has not included the cost of the gaming system in her graph, the graph is inadequate for the purposes of this problem. The student has not advanced through the problem in the manner that meets the teacher's overarching goal of progress.

In lines 7–19, the teacher launches into a didactic interchange with the student about her graph. This response reflects a variety of different goal shifts taking place. First, the

1 J Is this what you are talking about?
2 T Yeah, but you only have to pay for the system once.
3 J ok.
4 T Do you know what I'm saying?
5 J yes.
6 D So, when you...when we do that do you want to include a xxxx [inaudible]?
7 T hmm... yeah we want the total cost, not just the [teacher positions himself
8 next to D, back turned to J] cost per game. You know what I'm saying?
9 D == so, so ==
10 T See, this... [points to her graph] this is telling me how much zero games cost.
11 I want to know... when I look at the cost over here? I want to know the cost of a
12 system plus the games. This one is only telling me games.
13 It's not telling me anything...I know that if I'm only buying...
14 I already know that if I'm just buying the games right?
15 [squats down and looks at D] If I buy one game its certainly its cheaper to
16 buy Pretendo right? Because its very easy. Pretendo games cost forty dollars
17 and Sega games cost fifty. So I would never ever buy Sega, except why?
18 Except a Sega system is cheaper. So this graph [points to D's graph]
19 looks like Sega is always more expensive. But Sega is not
20 always more expensive. If I was only going to buy 3 games,
21 for instance right?, and I knew I was never going to buy
22 more than 3 games, which system should I buy?
23 D I don't know.
24 T See this won't tell me that. So these numbers over here should start out
25 with the cost of the system [flips student booklet to different page] ...180?
26 So for zero games, no games at all, it's going to be 180 here, 180 and 120. ok?
27 D So start with 120?
28 T yeah
29 D ... and then go up?
30 T and then 180, and then increase the amount for each of those games right?
31 So yeah, this would start, yeah, would start up here somewhere
32 and up here somewhere [points to D's graph].



Goal legend corresponding to the list of goals on the right-hand side above:

G1: Have students work together in group.
G2: Have student correct her graph.
G3: Give student a start on how to build the correct graph.

FIGURE 7. Transcript, goal parsing, and belief bundles for Case #2.

high-level collaborative goal (G1) of students working together is suspended. The teacher focuses all his attention and explanation on D. J and the other student sitting at the table are not included in the interaction. Moreover, the teacher does not invite D to use the other students as resources even though J's previous question was related to the same condition, the inclusion of the gaming system, that the teacher attributed to D's inaccurate graph.

With the collaborative goal suspended, a new goal (G2) emerges. The teacher wants to get the student to correct her graph. To realize the goal of getting the student to correct her graph, he employs a standard teaching routine. He poses a scenario to demonstrate that her graph is incomplete and inadequate for the purposes of this problem. This scenario culminates with a question to the student, "which system should I buy?" (Line 19).

The student responded to the teacher by saying, "I don't know" (line 20). The student's response triggers another goal shift. D's response confirms to the teacher that she doesn't yet understand what is wrong with her graph. At this point another goal emerges (G3) that relates to G2. The teacher wants to help the student start to build the correct graph. He shows her explicitly where she can find the information she needs in her instructional booklet. Then he proceeds to tell her where each line on her graph should start by pointing to the exact places on the *y*-axis (the total cost: US$180 and US$120, respectively). The student says very little during this interaction—only cautiously verifying where he wants her to start the graph. Once the teacher has told her how to correct her graph, he leaves the table.

The role of the belief bundle. The two initial goal shifts (after line 6 and line 23) were triggered when the teacher examined the students' progress on the solution. The student's solution attempt was not meeting his overarching progress goal, thus he had to provide assistance. However, he chose not to include the other students at the table. This exclusion of the other students is evidence of a suspension of his collaborative goal of wanting the students to work together. Concomitant with the suspension of G1 is the formulation of a new goal (G2) of getting the student to correct her graph.

With this shift in goals, a belief bundle becomes apparent (as shown to the right of the goal traces in Fig. 7). The beliefs in this bundle are nested in his standard theories of mathematics, teaching and learning. The focus on correcting the student's graph to be the appropriate mathematical representation is tied to his belief about mathematics as a set of procedures, formulas and equations. Her graph is not representative of the correct procedure or equation needed to solve the problem. The second belief in the bundle is connected to his standard belief about learning: students learn by listening to the teacher explain how to do a problem. Related to this learning belief is the complimentary standard teaching belief that it is important to explicitly tell students how to do a mathematics problem. All three of these beliefs form a belief bundle that became apparent at the shift in goals and explain his actions.

Another place where a shift in goals occurs is in response to the uncertainty the student expresses in line 23. The goal (G3) that emerges relates to the previous goal of getting the student to correct her graph. However, this goal is more local in character and focuses on explicitly telling the student the appropriate procedure to correct her graph. In this case, the belief bundle that is apparent at this shift remained intact from the initial goal shift. The same mathematics, learning and teaching beliefs are apparent in the formulation of the new goal G3. However, the standard teaching belief about the importance of explicitly telling students how to do a mathematics problem is emphasized in the belief bundle.

The existence of the belief bundle and the degree of activation of particular beliefs in that bundle influenced the formation of the new goals. In this case, Mr. Martin's standard beliefs about teaching and learning comprised the belief bundle. If his collaborative beliefs about teaching and learning had remained active we contend that a corresponding goal may have been formulated and illustrated in the teacher's actions in the form of verbally encouraging students to work together. Since this did not occur, as evidenced by the teacher actions during this episode, we assert that his attributed collaborative beliefs had low activation.

6. GENERAL DISCUSSION AND NEXT STEPS

The examples discussed in the previous section focused on the critical role of teacher beliefs in teacher practice. This section will provide a brief summary of the findings and an elaboration of the next steps of this empirical investigation.

Unlike previous investigations, our study provides insights about beliefs and their connection to goals and actions of teachers *during the act of teaching*. The fine-grained analysis and subsequent discussion of each example highlights how beliefs influence the formulation of particular goals. Specifically, the examination of shifts in the goals provides insight into which beliefs were playing a particularly influential role in the formulation of the new goals.

In our two examples, the teachers had particular beliefs about mathematics, teaching, learning and students. As the lesson unfolded, there were unanticipated instances that provoked changes in goals. For example, a student gave an explanation that did not use the particular algebraic formula being discussed during the lesson. For the teacher, this resulted in a new goal to assist the student to utilize the correct formula. This shift in goals revealed the belief bundle consisting of particular math, learning, and teaching beliefs that influenced the formulation of that new goal. Although we acknowledge that there are other beliefs influencing the action of the teacher, the belief bundle contains the combination of beliefs that provide a sufficient explanation for the formulation of the new goals and the observed teacher action.

The study also identified the beliefs about the nature of teaching, learning and mathematics that were especially influential on the teachers' practice. These beliefs manifested themselves at the particular goal shifts during the classroom interactions. One question to consider is how influential these beliefs are over time. If one belief appears to strongly influence the formulation of a goal, what is the mechanism and/or context that signals the prioritization of that goal? Does consistency and prioritization provide evidence of a particularly stable belief? In the example of Ms. Perry, her belief of mathematics as a collection of tools (formulas, procedures) for students to learn appears to strongly guide the direction of classroom discussions. Although one of her professed learning beliefs emphasizes that students learn by listening to each other, she clearly redirects students' attention to the formulas and procedures to be learned. Although the view of the nature of mathematics did not appear inconsistent with her beliefs about learning and teaching, how Ms. Perry thought about mathematics was a strong factor in formulating her pedagogical goals and her subsequent classroom actions.

The case is less clear with Mr. Martin. As noted, Mr. Martin was consciously trying to change his pedagogical practice to include group work. Therefore, his collaborative teaching and learning beliefs played a central role in the formulation of goals and subsequent actions during small-group activities. However, as the example illustrates, there were times when Mr. Martin's actions appeared consistent with his "standard" teaching beliefs (to explicitly tell the student how to do the mathematics problem) and his "standard" learning beliefs (students learn by listening to the teacher's explanation). His professed collaborative learning belief that students learn through discussion with each other appeared suspended during this interaction. Why did his standard beliefs manifest themselves so strongly in this interaction? What may have triggered this clear change in goals and actions? How influential are the attributed collaborative beliefs when evidence, such as the analysis above, strongly suggests the persistence of his standard beliefs in formulating pedagogical goals? Over time, will this apparent dual system of beliefs transform into a new hybrid containing elements of both, or will one prevail? More empirical research needs to be done to understand the nature of teacher beliefs and how they manifest themselves in the ongoing development of goals. We want to understand and be able to explain how this complex relationship evolves over time to further the current understanding of the cognitive development of teachers.

In addition, the methods used in these examples provide a new, more detailed way to examine the relationships among teacher beliefs, goals, and practice. We need to continue to refine the methods used to examine these relationships and to further understand the underlying mechanisms of this relationship. One promising path in this direction is the closer investigation of the belief bundle. This research identified the prevalence of particular attributed teacher beliefs and some potential pairings of specific beliefs revealed at certain goal shifts. The role of the belief bundle provides a hint at what might characterize the mechanism that determines how goals are shaped and reshaped by particular beliefs.

Our current understanding of belief bundles, as a component of mechanism, is limited. This may be due to the consistent nature of the belief bundle compositions in the particular episodes we studied. For instance, in both case examples, the beliefs bundled together and revealed during the goal shifts remained the same over the entire interaction analyzed. In the example of Ms. Perry, the belief bundles revealed during the goal shifts were comprised of three beliefs. These beliefs were about the nature of mathematics (collection of tools), the teaching of mathematics (it is important for students to have clear examples), and the learning of mathematics (students learn by listening to others). This particular composition appeared to influence the formulation of the goals and subsequent actions of this teacher. However, we need to continue the investigation to determine how much variation exists in the bundle composition in other episodes. On the other hand, the consistency of the bundles we identified may indicate the particularly strong influence of certain beliefs and for particular beliefs to tend to act in consort with each other. We are also interested in exploring whether particular beliefs tend to bundle in predictable ways.

As we look toward expanding our analysis to include an entire class lesson and perhaps a series of lessons, we expect to develop further insight about the bundle composition. Preliminary analysis of some additional episodes indicates that different beliefs comprise different belief bundles over the same episode of the interaction. We would like to explore additional episodes to track these different bundle compositions and document how these beliefs influence specific goal formulations and actions. In the course of our future research we hope to build on the findings of this study and gain insight into the underlying mechanism of the belief–goal connection in the teaching process.

Future work will also explore the use of these methods to examine a greater diversity of teaching situations. If we want to deepen our understanding about how beliefs and goals are related in the moment-to-moment teacher decision-making processes and actions, we must investigate different teachers in different contexts. One line of empirical investigation could be aimed at trying to model teachers teaching different courses. For example, if the same teacher taught a "transition" mathematics course for students needing foundational concept and skill building and an "honors" algebra class, would the same beliefs and their various bundle compositions be active given a similar unanticipated context? How would one characterize the relationship between beliefs and goals if the teacher actions were similar or dissimilar in the two classes? In addition, future analyses may focus on novice teachers to gain an understanding of how beliefs and goals are manifested developmentally over time. Do beliefs manifest themselves differently and/or consistently with teachers who are in the classroom for the first time? How might these beliefs and goals change through experience? Lastly, modeling teachers in transition—teachers making changes in their pedagogy and/or implementing novel curricula—may shed more light on how beliefs can be suspended, maintained or transformed, and affect the prioritization of goals and subsequent actions.

Potential uses for this type of work go beyond building a collection of case studies. The methods used in this work could also be used for teacher professional development. A single teacher's classroom practice could be analyzed and then the resulting representation of the classroom interaction could be the subject of a reflective pedagogical discussion. These representations could be used by teachers, researchers, and/or professional development people, or as the focus of attention in a teacher "video club" (Frederiksen et al., 1992). The intention would not be to evaluate the teacher's practice, but instead to provide a tool for focusing the discussion on the relationship between beliefs and practice. The process could promote and develop teachers' self-reflections on their own practice.

This study provides empirical evidence of how particular elements of teacher cognition, beliefs and goals work together to explain the moment-to-moment decisions and actions of teachers. It is a first step in examining this complex relationship. We contend that the fine-grained analysis of teaching practice provides a different level of explanation of how teachers make decisions. This type of work can also help us understand how those decisions and subsequent actions are shaped. By further investigating the particularly influential role beliefs play in the teaching process, we can obtain a better understanding of the teaching we see in the classroom.

NOTES

1. The teacher had made this suggestion to J a few minutes earlier while he was sitting at his desk filling out a disciplinary form.
2. Overarching goals are global forms of teachers' goals. They are always prevalent and active in the teaching process and are therefore not included in the goal parsing.

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