Module 1: Graph Theory

This material corresponds to chapters 1 and 2 of the textbook, *For All Practical Purposes*
GRAPH THEORY
CHAPTERS 1 AND 2

TIME FRAME: 13 days

ENDURING UNDERSTANDINGS:

There are many ways of representing information in mathematics, and carefully
choosing the form you use can greatly simplify the problem-solving process. Graphs of
the kind studied in this unit provide one such means.

ESSENTIAL (ASSESSMENT) QUESTIONS:

Chapter 1
1. What steps take a world pathway and abstract it to a circuit?
2. What are the criteria to eulerize a given path?
3. What effect does eulerizing have on a pathway event in the world?
4. What is the importance of valence in determining an Euler Circuit?

Chapter 2
1. What steps take a world pathway and abstract it to a circuit?
2. What are the criteria for creating a Hamiltonian Circuit?
3. What are the differences in criteria for Euler versus Hamiltonian Circuits?
4. How does one find a minimum time (critical path) Hamiltonian Circuit?

CRMS

Problem Solving/Reasoning: 1.1, 1.2, 1.3
Communication: 2.1, 2.2c, 2.3
Connections: 3.1b, 3.2a, 3.3b, 3.4a
Number Sense: 4.2a
Probability/Statistics: 6.1c, 6.2c

AT THE END OF THE MODULE STUDENTS WILL KNOW AND BE ABLE TO:

1. Use counting principles to determine the number of possible circuits.
2. Determine whether a graph has an Euler and/or Hamiltonian Circuit.
3. Describe differences between Euler and Hamiltonian Circuits.
4. Apply Kruskal’s algorithm to find optimal circuits.
5. Understand implications of Eulerizing a graph in a real world situation.
6. Solve real world problems with graph theory.
7. Analyze a graph to determine an optimal eulerization.
PREREQUISITE KNOWLEDGE/SKILLS: Concepts of odd and even.

PRE-ASSESSMENT: None

ACTIVITIES:

- Euler Circuit Activity worksheet
- Hamiltonian Circuit Activity worksheet

POST-ASSESSMENT: Navigating an Amusement Park project

LITERACY STRATEGIES INTRODUCED:

- Cornell Notes
- Concept Maps

RESOURCES:

- Applets from FAPP Instructor Resource CD-ROM
**DAILY PLAN**

**Day 1**
- Group activity: **Euler Circuit Activity** (Worksheets 1A, 1B, 1C).
- Have students take Cornell Notes on discussion of solutions (*though they won’t know that’s what it’s called until Day 2*).

**Day 2**
- Go over Concept Map for Graph Theory.
- Formally introduce the Cornell Note-Taking System.
- Discuss Euler Circuits.
- **Literacy: Guide to using the vocabulary and anticipation guide.**
- Assign homework exercises on terminology and Euler Circuits.
  - Select from: 7e  pp. 25-28 #1-30
  - 8e  pp. 22-25 #1-32

  **HW:** Read 7e  pp. 3-12 and take Cornell Notes
  8e  pp. 3-12 and take Cornell Notes

**Day 3**
- Review
- Discuss Eulerization.
- Assign homework exercises on Eulerizing a graph.
  - Select from: 7e  pp. 28-33 #31-57
  - 8e  pp. 25-28 #33-61

  **HW:** Read 7e  pp. 12-18 and take Cornell Notes
  8e  pp. 1-16 and take Cornell Notes

**Day 4**
- Begin Part I of **Navigating an Amusement Park.**

**Day 5**
- Group work: **Hamiltonian Circuit Activity** (Worksheets 2A, 2B, 2C)
- Have groups report out and take Cornell Notes looking for similarities and differences in solution approaches.

**Days 6-7**
- Discuss Counting Principles.
- Have students do applet 1 from FAPP Instructor Resource CD-ROM.
- Assign homework exercises for Hamiltonian Circuits.
  - Select from: 7e  pp. 64-69 #1-32
  - 8e  pp. 54-58 #1-34

  **HW:** Read 7e  pp. 36-43 and take Cornell Notes
  8e  pp. 31-37 and take Cornell Notes
Day 8
- Go over homework.
- Discuss Traveling Salesman Problem.
- Have students do applet 2 from FAPP Instructor Resource CD-ROM.
- Assign homework on the Traveling Salesman Problem.
  Select from: 7e pp. 69-72 #33-47
  8e pp. 54-58 #35-49
HW: Read 7e pp. 43-48
  8e pp. 43-46

Days 9-10
- Go over homework.
- Discuss Spanning Trees (Kruskal's algorithm).
- Have students do applet 3 from FAPP Instructor Resource CD-ROM.
- Assign homework for Spanning Trees.
  Select from: 7e pp. 72-76 #48-67
  8e pp. 60-63 #50-69
HW: Read 7e pp. 48-55
  8e pp. 43-46

Day 11
- Go over homework.
- Discuss Critical Path Analysis.
- Revisit Introductory Problems review student solution ideas.
- Assign homework on Critical Path Analysis.
  Select from: 7e pp. 76-77 #68-75
  8e pp. 64-64 #70-77
HW: Read 7e pp. 55-59
  8e pp. 46-52

Days 12-13
- Introduce Part II of Assessment Project.
- Give students time to work on project individually and to ask clarification questions.

Day 13
- Collect and share Assessment Projects.
EULER CIRCUIT ACTIVITY DESCRIPTION

These three worksheets (named 1A, 1B and 1C) are designed as a discovery exercise for students beginning the first chapter of *For All Practical Purposes*. The intention is that a class be divided into groups of 3-4 students and each group is given one of the three worksheets. Students should spend about 15 minutes trying to find Euler Circuits.

This activity is weaved together with an introduction to the Cornell Note-Taking System which should be formally introduced the next class day. At the end of the group work, students will share their observations with the class, and the results will be recorded on an overhead in a style that previews what they will see the next day. (The instructor is strongly encouraged to read in advance the description of Cornell Notes following this activity.)

Ideally, instructors will motivate this activity with a simple real-world example, such as a street cleaner trying to hit each street as efficiently as possible. The instructor should avoid introducing terminology like “valence” and “eulerizing a circuit” until after the students have worked on the activities.

After 15 minutes, the teacher can select one student from each group to report on their groups’ findings to the rest of the class. It might be useful to have additional copies of the worksheets printed on overhead transparencies to make it easier for students to explain their findings.

Afterward, the instructor should provide closure to this activity by making sure that students have seen the key idea: each vertex must have an even number of edges meeting there. That would be the appropriate time to introduce some of the relevant terminology.

**SUGGESTED TIME**

- 5 minutes for assembling groups and handing out worksheets
- 15 minutes for students to work in groups
- 15 minutes for sharing-out
IDEAS FOR RELATED PROJECTS

Some groups may finish the activity very quickly. Here are some ideas for additional activities that can be used to for extended learning:

- Ask students to draw a map of their school, convert it to a graph, and determine if a custodian waxing the floors could hit each hallway and stairwell exactly once and finish where he started.
- Find a street map of a local city or town from the internet. Have the students convert it to a graph and determine if a street cleaner can hit each street exactly once and finish where he began.

TIPS ON SHARING-OUT

After the students finish working in groups, the teacher will ask students to share their results as well as their processes with the rest of the class. The teacher should consider recording these results on an overhead or document camera in the style of Cornell Notes. The teacher does not need to talk about Cornell Notes at this stage – it will be enough to produce an example of the format that can be referred back to when Cornell Notes are formally introduced on Day 3 (so saving the transparency would be a good idea).

MATERIALS

The next few pages are the worksheets – there are three of them, and each has two sides. The teacher may decide whether to give each student her own worksheet or to provide only one worksheet for each group.

After the worksheets, you will find a sample of the kinds of responses we expect from students during the sharing-out period following group work. This sample is not intended to be presented to the students – it is simply an illustration of what we imagine will come from group discussion.
Euler Circuit Activity Solutions

All three versions of the worksheet have the same front side but different backsides.

Note on Terminology: The answers recorded below make use of the term “Euler Circuit” which will not have been introduced to students yet at the time of this activity. These solutions are for the teacher’s reference only.

Front Side:
1) No Euler Circuit
2) No Euler Circuit
3) No Euler Circuit
4) No Euler Circuit
5) Euler Circuit
6) Euler Circuit

Back Side:
Version A:
7) No Euler Circuit
8) Euler Circuit

Version B:
7) Euler Circuit
8) No Euler Circuit

Version C:
7) Euler Circuit
8) No Euler Circuit

Comments
The only difference between the final two graphs on each worksheet is the presence of an edge in one graph that is not in the other. In one case (Worksheet 1A), the additional edge makes the graph not have an euler circuit; in Worksheets 1B, the additional edge eulerizes the graph. In Worksheet 1C, it looks like an edge has been added in one place and deleted in another. (Deleting an edge is not a legal step in eulerizing a graph.) The instructor may wish to take advantage of these observations in a later class when the terminology is introduced.
SAMPLE NOTES FOR SHARING-OUT AFTER EULER CIRCUIT ACTIVITY  
(Formatted in the Cornell Notes Style)

Divide an overhead transparency into three sections as shown below. Then write down in the large, top right box the students’ observations from the activity. Afterward, come up with “cue questions” or keywords for the top left box that would remind students of the material on the right. Finally, come up with a short summary for the bottom box.

<table>
<thead>
<tr>
<th>When could every edge be traced exactly once?</th>
<th>You can only trace each edge exactly once when every vertex has an even number of edges meeting there.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>It was always possible to cover every edge, but you might have to traces some edges more than once.</td>
</tr>
<tr>
<td></td>
<td>Other times you could trace every edge once, but not end up where you started.</td>
</tr>
<tr>
<td>How can adding an edge affect the graph?</td>
<td>Some of the graphs that could be traced exactly once were a lot like graphs that couldn’t, but had an extra edge added.</td>
</tr>
<tr>
<td></td>
<td>Other times adding an edge made it not possible to trace every edge exactly once.</td>
</tr>
</tbody>
</table>

Summary: Observations about tracing edges of graphs during the first group worksheet activity.
Graph Theory - Worksheet 1A

For each graph, determine if there is a path through all the vertices (the black dots) along all the edges (the line segments) that ends in the same place it begins and that traces every edge exactly once. (Note: it is okay to pass through a vertex more than once, but not an edge.)
Graph Theory - Worksheet 1B

For each graph, determine if there is a path through all the vertices (the black dots) along all the edges (the line segments) that ends in the same place it begins and that traces every edge exactly once. (Note: it is okay to pass through a vertex more than once, but not an edge.)
Graph Theory - Worksheet 1C

For each graph, determine if there is a path through all the vertices (the black dots) along all the edges (the line segments) that ends in the same place it begins and that traces every edge exactly once. (Note: it is okay to pass through a vertex more than once, but not an edge.)
INTRODUCTION TO CONCEPT MAPS

The following page is a sample of a Concept Map – a literacy strategy that aids some students in learning by helping them to sense an overall picture of the course content.

*Global Learners* – students who like to see a whole picture before they can successfully focus on more detailed descriptions – can benefit from being shown such an overview in advance of the main content. (This is in contrast to *Sequential Learners* who are happy to take each lesson one-at-a-time and wait until the end to see how it all fits together.)

Later in the course, students will be asked to create their own Concept Maps from scratch as a method of reviewing material they have already learned. At this beginning stage, however, we will simply be illustrating the construction of such a map for the students as a way of previewing material to come.

We have also integrated the Concept Map with other overview-type devices: lists of relevant homework problems, new vocabulary words and self-test questions. Together, these resources should help both the instructor and students to organize the entire unit in their minds.

USING THE SAMPLE CONCEPT MAP

The map that follows is a sample of the map a teacher might create for her students. This map should not be copied and handed to students. Instead, the instructor should create such a map on an overhead transparency or on the board, and students should copy it onto their own papers. Of course the instructor may wish to keep this sample map at hand as a reference during this activity, but she should not feel bound to duplicate its contents verbatim.

The instructor will find a blank concept map in the appendix of this manual. She may make copies of it to distribute to students, or she may expect students to create their own drawings on blank paper.

The teacher should try to model the process of creating a concept map in such a way that the students will eventually be able to create their own concept maps after working through a chapter. Global learners may find these to be useful study aids.
**CURRENT UNIT**
Management Sciences: Graph theory

**LAST UNIT / Experience**

**NEXT UNIT / Experience**

<table>
<thead>
<tr>
<th>Unit / Homework Schedule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler Circuits and Paths</td>
</tr>
<tr>
<td>Chinese Postman Problem</td>
</tr>
<tr>
<td>Urban Transversal Problems and digraphs</td>
</tr>
<tr>
<td>Hamiltonian Circuits</td>
</tr>
<tr>
<td>Traveling Salesman Problem</td>
</tr>
</tbody>
</table>

Is about...

A finite set of points and connecting links, which are used to find the best method for solving a problem (optimal solution)

Helps with...

- Critical Path Analysis
- Minimum Cost Spanning Trees

Leads to...

1. What are the differences and similarities between Hamiltonian and Euler circuits?
2. Is it possible (why or why not) for a graph to have all even valences and NOT have an Euler circuit?
3. How does the fundamental counting principal apply to Hamiltonian circuits?
4. Would there ever be a reason to find minimum-cost spanning tree for a weighted graph in which the weights on some of the edges were negative?
INTRODUCTION TO CORNELL NOTES

SUGGESTED TIME

10 minutes before lecture
5 minutes after lecture
5 minutes at the beginning of the next class reviewing Cue Questions

DESCRIPTION

The teacher should briefly explain the ideas of the Cornell Note-taking System described on the next page. She may also wish to produce copies of this description to hand out to the students.

Then the students will immediately practice using this system during a lecture on the material from Chapter 1 of the textbook. The teacher should ask students to divide their note pages in the three-box style and to just use the large box on the right for taking notes. After the lecture, the teacher should look over a few students’ notes and use them as examples to show the class how to write cue questions. Students should spend a few minutes working individually at the end of class writing cue questions for the notes they took.

Students should be asked to take notes and to come up with cue questions from the suggested reading for homework so that they can be shared at the beginning of the next class.
Cornell Note-Taking System

One of the College Readiness Mathematics Standards is the following student attribute: “Takes responsibility for own learning.” To do that effectively, students must take good notes from lectures, class discussions and reading their textbooks, but this note-taking skill is not something that comes naturally to all students – it must be learned.

To that end, we have incorporated the Cornell Note-taking System into the design of this course. This system, also called Cornell Notes, was originally devised in the 1950s by Walter Pauk, an education professor at Cornell University, and publicized in his best-selling book How to Study in College. It has been adapted in various form, one of which we present here.

The note-taker divides a page into 4 sections, as shown in the figure. The top region is for Heading information: Course, Name, Date. A student could also include a chapter title or lecture topic in the area.

The section labeled Notes in the figure is for the actual taking of notes during a lecture or class discussion or while reading the textbook.

The section labeled Cue Questions is used later, after the notes have been taken. A student writes short questions here whose answers can be found in the notes on the right side of the paper. Then when a student is studying, she covers the notes on the right side with another sheet of paper and tries to answer the questions on the left. Rather than writing a question, a student can also write a key word or term whose definition appears in the notes on the right side.

Finally, the short section at the bottom is for students to write a Summary of what the current page of notes contains. This summary should serve two purposes: to help students understand the notes they have written by explaining them in a concise fashion, and to give the students a quick way to review this page at a later date.

This style of recording notes and reviewing them later to write Cue Questions has proven to be a very effective means of organizing and studying for almost any academic subject- not just mathematics!
HAMILTONIAN CIRCUIT ACTIVITY DESCRIPTION

SUGGESTED TIME

3 minutes to organize groups and hand out worksheets
15 minutes of group work
15 minutes of sharing out and modeling Cornell Notes
Remaining class time can be used for lecture on Chapter 2 terminology

GROUP ACTIVITY DESCRIPTION

Students will work in groups of 3-4, with each group working on one of the worksheets provided on the following pages. Ideally, for each worksheet, there will be two separate groups using it, so that during the sharing-out period at the end, one group will be able to check the other’s results and possibly offer alternative approaches for finding the solutions.

Again, it is recommended that the teacher record (or have a student record) the key points from sharing-out on an overhead transparency in Cornell Notes fashion to reinforce the usefulness and practice the method of this system. Students should then be encouraged to continue using the Cornell Notes system during the lecture in the following class, and to write Cue Questions along side their notes for homework.
HAMiltonian Circuit Activity Solutions

And Additional Commentary

This exploratory exercise is designed to trigger the following kinds of questions in students:

1) Is there a solution (fastest drop off of fellow partiers, minimum boating time for a fishing contest or a least expensive tour with the relatives) to a life problem?
2) Is the solution unique?
3) Is there a good strategy to produce the unique solution or the solutions?
4) Is there a best strategy?

Teachers should try to avoid asking these questions themselves and instead to guide students toward coming up with such questions.

These worksheets should also motivate the need for some vocabulary that will make it easier to discuss the ideas involved. Teachers should look for opportunities to insert the vocabulary during the sharing-out that follows the activity when students start talking about an idea for which we already have a term (for example, “There’s a term for what you’re describing: it’s called a Hamiltonian Circuit”). Thus we try to pair definitions with concepts the students have already discovered in their group discussions.

Watch for ways to compare student work to the heuristic algorithms: the nearest-neighbor algorithm and the sorted-edges algorithm. Both of these algorithms are greedy in the sense that each time a choice is made, they make the choice that seems best in the short term.

There are three worksheets. A class of 24 would, ideally, have six groups of four students, and each worksheet would be worked on by two of the groups. The idea is to establish the concept of “check” – whenever one group reports an observation, the other
group should be asked if that’s consistent with their findings. Every claim made during the sharing-out should be checked in this manner if possible.

**SOLUTION TO WORKSHEET A**

*FMLEF* is quickest and takes 36 minutes. Student approaches may vary. Have each group critique the work of the other group doing the same worksheet to check for errors and to look for alternative approaches. The *FMLEF* answer is the sorted edges algorithm or the nearest neighbor starting from *L*, *M* or *E*. The nearest neighbor starting from *F* yields 38 minutes.

**SOLUTION TO WORKSHEET B**

*ABMCA* is both the nearest neighbor and sorted edges circuit. The total time to travel this circuit is 246 minutes. Yet the most efficient would be *MBCAM*. The total travel time is 244 minutes. The *MEBCAM* circuit is obtained by using the nearest neighbor algorithm starting at *B*. It gives the greatest amount of time to fish.

**SOLUTION TO WORKSHEET C**

*SeTPVSspSe* is the least expensive tour using nearest neighbor tour and costs $463; *SeSpPVTSse* is the least expensive sorted edges tour and costs $464.

In all cases, watch for student work that does the algorithms without formal instruction so that you can build on those intuitions when you begin defining the terminology in this chapter.
Graph Theory – Worksheet 2A

Determine the route that will allow Francine to drop off her friends Maria, Latecia and Eva after a party at her house in the least time possible. A picture of the routes between the friends’ houses and the travel times along each route are shown below. Each circle represents a house, and the letter indicates whose house it is (for example, the circle labeled M is Maria’s house). Each line segment indicates a travel route, and the number listed by it indicates the travel time along that route (in minutes). Note that Francine must return home at the end of the trip.

As you determine the least-time solution, keep track of the different approaches your group tries, including those that succeed and those that fail. You will be expected to report out both successful and unsuccessful approaches. As a class, we will look for a pattern in the various groups’ solutions.
Graph Theory – Worksheet 2B

Determine the route that will allow Maria to efficiently fish at three “hot spots”, represented by the circles labeled A, B and C in the figure below, in her bass-fishing championship.

Maria wants to spend the most time possible fishing rather than traveling between the hot spots, but she must fish at all three spots. Also, she must start at the position labeled M, and she must return there at the end for a “weigh-in” to determine the champion. Each line segment in the figure indicates a travel route, and the number listed by it indicates the travel time along that route (in minutes).

As you determine the solution, keep track of the different approaches your group tries, including those that succeed and those that fail. You will be expected to report out both successful and unsuccessful approaches. As a class, we will look for a pattern in the various groups’ solutions.
The figure below indicates five major cities in the Pacific Northwest, represented by circles, and train routes between the cities, represented by the line segments. The number along each line segment indicates the cost of a train ticket along that route.

Determine the route which will allow you to visit all five cities, starting and ending at Seattle, for the lowest possible price.

As you determine the solution, keep track of the different approaches your group tries, including those that succeed and those that fail. You will be expected to report out both successful and unsuccessful approaches. As a class, we will look for a pattern in the various groups' solutions.

Legend:

P=Portland, Se=Seattle, Sp=Spokane
T=Tri Cities, V=Vancouver (Canada)
These reading guides are optional and are designed to assist student. The college reading level of the textbook is challenging.

Anticipation Guides were developed to appraise prior knowledge at the prereading stage and evaluate the acquisition of content based on postreading responses (Reaction Guides) to the guide questions. Since Anticipation Guides encourage a personal, experience based response, they serve as ideal springboards for large and small group discussions. (Head & Readence, 1992)

Anticipation Guides consist of the following steps:

**Planning:** Select major concepts and supporting details in a text selection, lecture, or other information source. Identify students’ experiences and beliefs that will be challenged and, in some cases, supported by the material. Create statements that reflect students’ prereading beliefs and that may challenge and modify those beliefs. Three to five statements are usually adequate. Arrange the statements on paper, transparency, or board.

**Prereading:** Have students respond to each statement individually. You may ask them to justify their responses for a reference point during a later discussion. Engage the students in a prereading discussion asking them to justify their responses to the statements.

Notes: You may include an “I’m not sure” response, for students who do not feel comfortable with a definite answer. This will help determine the students’ prior knowledge. Let the students know the statements are designed to make them think about topics and to make them think about what they will be learning. (Literacy and Learning: Reading in the Content Areas, 16)

For the purposes of this class, give the students the vocabulary preview and the anticipation guide prior to reading a certain section of the book. Have them fill them out individually in class. You can facilitate a brief discussion as a whole class or small group discussion on what their prior knowledge is concerning the content.

These activities will help the students’ access prior knowledge before they read the text as well as give them a preview to the concepts they will be reading.

There is an extra blank for you to fill in your own statements for each section or you can have each student create their own to be shared with the class.
## 1.1 Euler’s Circuits

### Vocabulary Preview

<table>
<thead>
<tr>
<th>Terms</th>
<th>??</th>
<th>?</th>
<th>!</th>
<th>Written Definition</th>
<th>Logograph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operations research</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimal Solution</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Graph</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertex (vertices)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Edges</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Path</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Circuit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Euler Circuit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Key:**
- ?? I have NO idea what this means
- ? I have heard it before…but I’m not sure
- ! I know this word! It means…

**Logograph:** Sketch what your mind “sees” when you read each word.
**ANTICIPATION GUIDE**

**INSTRUCTIONS:**
Read each statement and write **Agree** in the blank if you believe the statement and could support it or put **Disagree** in the blank if you do not believe it or could not support it. After you finish reading the selection – we will revisit this and check the validity of each statement.

<table>
<thead>
<tr>
<th>Before Reading</th>
<th>After Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td>The only <strong>graphs</strong> in math are those that display numerical relationships, like coordinate and bar graphs.</td>
<td></td>
</tr>
<tr>
<td>Math can help save time for traversing routes; such as delivering mail, removing snow, and checking parking meters.</td>
<td></td>
</tr>
<tr>
<td>An <strong>Euler circuit</strong> is a <strong>circuit</strong> that covers each <strong>edge</strong> of a graph once and only once.</td>
<td></td>
</tr>
</tbody>
</table>
1.2 Finding Euler’s Circuits

Vocabulary Preview

<table>
<thead>
<tr>
<th>Terms</th>
<th>??</th>
<th>?</th>
<th>!</th>
<th>Written Definition</th>
<th>Logograph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valence</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Connected Graph</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Key:  ??  I have NO idea what this means
       ?  I have heard it before…but I’m not sure
       !  I know this word! It means…

Logograph: Sketch what your mind “sees” when you read each word.
### ANTICIPATION GUIDE

**INSTRUCTIONS:**
Read each statement and write **Agree** in the blank if you believe the statement and could support it or put **Disagree** in the blank if you do not believe it or could not support it. After you finish reading the selection – we will revisit this and check the validity of each statement.

<table>
<thead>
<tr>
<th>Before Reading</th>
<th>After Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td>There is not a way to tell if a <strong>graph</strong> has an <strong>Euler circuit</strong> other than trial and error.</td>
<td></td>
</tr>
<tr>
<td>A <strong>graph</strong> is said to be <strong>connected</strong> if every <strong>vertex</strong> is connected to each vertex.</td>
<td></td>
</tr>
<tr>
<td>A <strong>valence</strong> is part of a window curtain.</td>
<td></td>
</tr>
</tbody>
</table>
1.3 **BEYOND EULER CIRCUITS**

**Vocabulary Preview**

<table>
<thead>
<tr>
<th>Terms</th>
<th>??</th>
<th>?</th>
<th>!</th>
<th>Written Definition</th>
<th>Logograph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chinese Postman Problem</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eulerizing a graph</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rectangular network</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Key:**
- ??  I have NO idea what this means
- ?  I have heard it before…but I’m not sure
- !  I know this word! It means…

**Logograph:** Sketch what your mind “sees” when you read each word.
## ANTICIPATION GUIDE

**INSTRUCTIONS:**
Read each statement and write *Agree* in the blank if you believe the statement and could support it or put *Disagree* in the blank if you do not believe it or could not support it. After you finish reading the selection – we will revisit this and check the validity of each statement.

<table>
<thead>
<tr>
<th>Before Reading</th>
<th>After Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td>The type of problem in which we want to minimize the length of a circuit by carefully choosing which edges to retrace is call the <strong>Chinese Postman problem</strong>.</td>
<td></td>
</tr>
<tr>
<td>In <strong>Eulerizing a graph</strong>, you make all <strong>valences</strong> odd.</td>
<td></td>
</tr>
<tr>
<td>There is no good way to <strong>Eulerize</strong> a <strong>rectangular network</strong>.</td>
<td></td>
</tr>
</tbody>
</table>
# 1.4 Urban Graph Traversal Problems

## Vocabulary Preview

<table>
<thead>
<tr>
<th>Terms</th>
<th>??</th>
<th>?</th>
<th>!</th>
<th>Written Definition</th>
<th>Logograph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Digraph</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Key:**
- ?? I have NO idea what this means
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- ! I know this word! It means…

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## ANTICIPATION GUIDE

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<table>
<thead>
<tr>
<th>Before Reading</th>
<th>After Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td>One way streets make it almost impossible to optimize a route.</td>
<td></td>
</tr>
<tr>
<td>A <strong>digraph</strong> helps represent the direction that can be traveled down a street.</td>
<td></td>
</tr>
<tr>
<td>An Israel Electric Company reduced the amount of routes to check the electric meters by 40% when they applied the math from Chapter 1.</td>
<td></td>
</tr>
</tbody>
</table>
# 2.1 Hamiltonian Circuits

## Vocabulary Preview

<table>
<thead>
<tr>
<th>Terms</th>
<th>??</th>
<th>?</th>
<th>!</th>
<th>Written Definition</th>
<th>Logograph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamiltonian Circuit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Edge weight</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum-cost Hamiltonian Circuit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Algorithm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Complete graph</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fundamental principle of counting</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Key:**
- ?? I have NO idea what this means
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- ! I know this word! It means…

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# ANTICIPATION GUIDE

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Read each statement and write **Agree** in the blank if you believe the statement and could support it or put **Disagree** in the blank if you do not believe it or could not support it. After you finish reading the selection – we will revisit this and check the validity of each statement.

<table>
<thead>
<tr>
<th>Before Reading</th>
<th>After Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A Hamiltonian circuit of a graph</strong> visits each <strong>vertex</strong> of the graph once and only once returning to the original vertex.</td>
<td></td>
</tr>
<tr>
<td>Every graph has a <strong>Hamiltonian Circuit</strong>.</td>
<td></td>
</tr>
<tr>
<td>The <strong>method of trees</strong> is always an easy way to determine a <strong>minimum cost Hamiltonian Circuit</strong>.</td>
<td></td>
</tr>
</tbody>
</table>
## 2.2-2.3 Traveling Salesman Problem

### Vocabulary Preview

<table>
<thead>
<tr>
<th>Terms</th>
<th>??</th>
<th>?</th>
<th>!</th>
<th>Written Definition</th>
<th>Logograph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traveling Salesman Problem</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nearest Neighbor Algorithm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Greedy Algorithm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sorted Edges</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heuristic Algorithm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NP-Complete Problems</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Before Reading</th>
<th>After Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimizing the cost of picking students up with a school bus at various stops is an example of the <strong>Traveling Salesman Problem</strong>.</td>
<td></td>
</tr>
<tr>
<td>Solving a <strong>minimum cost Hamiltonian circuit</strong> is called the <strong>Traveling Salesman Problem</strong>.</td>
<td></td>
</tr>
<tr>
<td>The <strong>nearest neighbor algorithm</strong> is an example of a <strong>greedy algorithm</strong>.</td>
<td></td>
</tr>
<tr>
<td><strong>Sorted edges algorithm</strong> begins by sorting the edges in decreasing cost or distance.</td>
<td></td>
</tr>
</tbody>
</table>
# 2.4 Minimum Cost Spanning Tree

## Vocabulary Preview

<table>
<thead>
<tr>
<th>Terms</th>
<th>??</th>
<th>?</th>
<th>!</th>
<th>Written Definition</th>
<th>Logograph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kruskal’s Algorithm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spanning Tree</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum Cost Spanning Tree</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Key:**

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<table>
<thead>
<tr>
<th>Before Reading</th>
<th>After Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Kruskal’s Algorithm</strong> deals with problems that involve graphs that don’t require circuits.</td>
<td></td>
</tr>
<tr>
<td>Just like the algorithm used in finding the <strong>minimum cost Hamiltonian circuit</strong>, <strong>Kruskal’s Algorithm</strong> does not guarantee an optimal result.</td>
<td></td>
</tr>
</tbody>
</table>
NAVIGATING AN AMUSEMENT PARK DESCRIPTION
Assessment Project

This project should be presented to students in two parts. Part I can be discussed on Day 4, at which time the lessons on Chapter 1 should be essentially complete. Part II will be discussed on Day 10, after the material on Hamiltonian circuits has been completed. Students will have time to begin these projects in class but they are expected to work individually on them at home and to submit the entire project (Parts I and II combined) on Day 11.

GRADING AND EXPECTATIONS

The following grading suggestions are meant to help you decide how to assess the students’ work. You should feel free to modify it if you feel something else is appropriate.

Part I:

1. (30%) Make a graph of all of the rides/attractions
   • Transfer each attraction to a point on a graph
   • Label each attraction with letters
   • Create a “Key” naming each attraction
   • Connect all points
2. (10%) Describe the valence of the graph
   • For each vertex, determine the valence
3. (30%) Create an Euler Circuit of the rides/attractions
   • Add edges to make all valences even.
4. (30%) What effect will this have on a guest at the park?
   • Discuss paths the guests will have to take as a result of Eulerizing the graph of the park
   • What is the objective for creating an Euler Circuit for the guests?

Part II:

1. (30%) Make a graph of all of the rides/attractions.
   • Transfer each attraction to a point on a graph
   • Label each attraction with letters
   • Create a “Key” naming each attraction
   • Connect all points
2. (45%) Create Hamiltonian Tours of the rides/attractions
• Use the Nearest Neighbor Algorithm
• Use the Sorted-Edges Algorithm

3. (25%) Estimate the minimum it would take to ride each attraction at Disneyland.
   • Describe the tour with the minimum time that you found in the previous steps.
   • Indicate how long that tour would take including the time it takes for each ride.

Solutions for the assessment will not be unique, but a sample solution is included after the 5-page handout.
ASSessment Project – Partial Solutions

Since solutions to the project may vary while remaining valid, depending on certain choices the students make, we do not provide a comprehensive solution for this project. However, there is a sample graph on the following page, and the valences for the graph are listed below:

<table>
<thead>
<tr>
<th>Attraction (Vertex)</th>
<th>Valence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Splash Mountain</td>
<td>5</td>
</tr>
<tr>
<td>Indiana Jones Adventure</td>
<td>2</td>
</tr>
<tr>
<td>Tarzan’s Treehouse</td>
<td>5</td>
</tr>
<tr>
<td>Mark Twain Riverboat</td>
<td>6</td>
</tr>
<tr>
<td>Buzz Lightyear Astro Blasters</td>
<td>4</td>
</tr>
<tr>
<td>Space Mountain</td>
<td>2</td>
</tr>
<tr>
<td>Alice in Wonderland</td>
<td>4</td>
</tr>
<tr>
<td>King Arthur Carrousel</td>
<td>4</td>
</tr>
<tr>
<td>Mad Tea Party</td>
<td>4</td>
</tr>
<tr>
<td>Pirates of the Caribbean</td>
<td>4</td>
</tr>
</tbody>
</table>
**NAVIGATING AN AMUSEMENT PARK**

*Graph Theory Assessment Project*

You visit Disneyland one day and would like to ride all the attractions listed in the table below.

<table>
<thead>
<tr>
<th>Attraction</th>
<th>Time to wait and to ride</th>
</tr>
</thead>
<tbody>
<tr>
<td>Splash Mountain</td>
<td>33 minutes</td>
</tr>
<tr>
<td>Indiana Jones Adventure</td>
<td>1 hour 5 minutes</td>
</tr>
<tr>
<td>Tarzan’s Treehouse</td>
<td>17 minutes</td>
</tr>
<tr>
<td>Mark Twain Riverboat</td>
<td>28 minutes</td>
</tr>
<tr>
<td>Buzz Lightyear Astro Blasters</td>
<td>45 minutes</td>
</tr>
<tr>
<td>Space Mountain</td>
<td>1 hour 25 minutes</td>
</tr>
<tr>
<td>Alice in Wonderland</td>
<td>38 minutes</td>
</tr>
<tr>
<td>King Arthur Carrousel</td>
<td>24 minutes</td>
</tr>
<tr>
<td>Mad Tea Party</td>
<td>38 minutes</td>
</tr>
<tr>
<td>Pirates of the Caribbean</td>
<td>1 hour 45 minutes</td>
</tr>
</tbody>
</table>

You also estimated the walking times between various attractions ahead of time by reading a map of the park, and those estimates are indicated in the table on the next page.

Use the information in these tables to answer the questions in each part of this project. *(Note: There is also a map of Disneyland included in this handout that you might find useful for placing vertices in your graph.)*
<table>
<thead>
<tr>
<th>Attraction</th>
<th>Buzz Lightyear Astro Blasters</th>
<th>Indiana Jones Adventure</th>
<th>King Arthur Carousel</th>
<th>Mad Tea Party</th>
<th>Mark Twain Riverboat</th>
<th>Pirates of the Caribbean</th>
<th>Space Mountain</th>
<th>Splash Mountain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alice in Wonderland</td>
<td>3</td>
<td>18</td>
<td>22</td>
<td>10</td>
<td>12</td>
<td>11</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>Tarzan's Treehouse</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Buzz Lightyear Astro Blasters</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Indiana Jones Adventure</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>King Arthur Carousel</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mad Tea Party</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mark Twain Riverboat</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pirates of the Caribbean</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Space Mountain</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Splash Mountain</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
DIRECTIONS

The directions below are split into two parts so that you can begin each part after you have learned the corresponding material in class. Your teacher will give you the dates to fill in on the following lines:

Date to start Part I: ____________________
Date to start Part II: ____________________
Date to submit completed project: ________________

PART I – EULER CIRCUITS

1. Make a graph of all of the attractions and the paths between them that are described in the given tables.
2. Describe the valence of the graph.
3. Create an Euler Circuit of the rides/attractions. (Note that you may need to eulerize the graph!)
4. Describe in a few sentences what the value of knowing this circuit would be to you at the park. Also explain what the real-world meaning is if you had to eulerize the graph.

PART II – HAMILTONIAN CIRCUITS

1. Make a graph of all of the attractions and paths connecting them; include walking times along the edges, from the information given in the tables.
2. Use each of the algorithms discussed in the textbook to find an efficient (in terms of time) circuit of the rides.
3. Compare the results of the algorithms you applied; what circuit takes the least amount of time?
4. Estimate the minimum time you would need at Disneyland to ride each attraction listed in these tables. (Note: You need to include the waiting/riding times in addition to the total walking time.)
Sample Graph of Attractions

Module 1: Graph Theory - Page 60
Given a map of Disneyland, create the following:

1. Make a graph of all of the rides/attractions.
2. Describe the valence of the graph.
3. Create an Euler Circuit of the rides/attractions. (Eulerize the graph if it’s needed.)
4. What effect will this have on a guest at the park?

Rubric for Grading Part I:

<table>
<thead>
<tr>
<th>Make a graph of the rides/attractions.</th>
<th>Each attraction is a vertex and is connected when possible to other vertices by edges. 10 points</th>
<th>Each attraction is labeled. 10 points</th>
<th>A key identifies each attraction. 10 points</th>
<th>/30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Describe the valence of the graph.</td>
<td>Each vertex is labeled with its valence 10 points.</td>
<td></td>
<td></td>
<td>/10</td>
</tr>
<tr>
<td>Create an Euler Circuit of the attractions.</td>
<td>Vertices are identified that require “Eulerizing”. 10 points</td>
<td>Edges are added to Eulerize the graph. 10 points</td>
<td>An Euler Path is identified. 10 points</td>
<td>/30</td>
</tr>
<tr>
<td>How will the Eulerized graph affect a guest at the park?</td>
<td>Paths a guest would take as a result of Eulerizing the park are discussed. 10 points</td>
<td>The objective for creating an Euler Circuit is stated. 10 points</td>
<td>Discussion includes the benefit of creating an Eulerized path 10 points</td>
<td>/30</td>
</tr>
</tbody>
</table>
ASSESSMENT CHAPTER 2:

Part II

Given a map of Disneyland, create the following:

1. Make a graph of all of the rides/attractions and paths connecting them; include walking times along the edges, from the information given on the tables.
2. Use each of the algorithms discussed in the text to find an efficient circuit of the rides.
3. Compare the results of the algorithms you applied; what circuit takes the least amount of time?
4. Estimate the minimum time you would need at Disneyland to ride each attraction listed in these tables. (You will need to include the wait/riding times in addition to the total walking time.)

Rubric for Grading Part II:

<table>
<thead>
<tr>
<th>Make a graph of the rides/attractions.</th>
<th>Each attraction is a vertex and is connected when possible to other vertices by edges.</th>
<th>Each attraction is labeled.</th>
<th>A key identifies each attraction.</th>
<th>/30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Create Hamiltonian Circuit of the rides/attractions.</td>
<td>The Hamiltonian Circuit created by the Nearest Neighbor Algorithm is identified.</td>
<td>The Hamiltonian Circuit created by the Sorted Edges Algorithm is identified.</td>
<td></td>
<td>/40</td>
</tr>
<tr>
<td>Estimate the minimum time needed to ride each attraction at Disneyland.</td>
<td>The tour with the minimum time is identified for both the Nearest Neighbor and Sorted Edges Algorithms.</td>
<td>The total time including the ride time for each circuit is indicated.</td>
<td></td>
<td>/30</td>
</tr>
</tbody>
</table>